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A SHORT COURSE IN MODEL DESIGN

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The growing enthusiasm for the use of computer models as aids to urban planning and administration derives less from the proven adequacy of such models than from the increasing sophistication of professional planners and a consequent awareness of the inadequacy of traditional techniques. As Lowdon Wingo has put it, planners are now prisoners of the discovery that in the city everything affects everything else:

In the good old days we tackled the slum in a straightforward way by tearing it down. Now we know the slum to be a complex social mechanism of supportive institutions, of housing submarkets, of human resources intertwined with the processes of the metropolitan community as a whole. . . . To distinguish favorable policy outcomes from unfavorable ones is no longer a simple matter. Decisions by governments, firms, and individuals in metropolitan areas turn on the state of such interdependent spatial systems as use of recreation facilities, transportation and communication nets, and the markets for land, housing, and even labor, rather than on the highly localized consequences directly elicited by policy actions. The rapid evolution of a genus of mathematical techniques, or models, to conditionally predict certain locational aspects of the behavior of urban populations has been both cause and consequence of these developments.[1]

. . . During the coming decade, it is safe to predict, many of our readers will be called upon to evaluate proposals for such models, or to participate in their construction. In this essay, I hope to provide some orientation to the model builder's way of thinking, interpret the jargon of his trade, and suggest a few standards for the evaluation of his product.[2]

Granted the complexity of the urban environment and the potentially extensive ramifications of planning decisions, we may ask, first of all, how computer models improve the planner's ability to generate sound policy and effective programs. The answer is certainly not that computers are wiser than their masters, but rather that they perform the most monotonous and repetitive tasks at high speed and

Reprinted from *Journal of the American Institute of Planners*, **31** (1965), 158–165, by permission of the author and editor.

with absolute mechanical accuracy. The model builder can make use of this capacity only insofar as he is able to perceive repetitive temporal patterns in the processes of urban life, fixed spatial relationships in the kaleidoscope of urban form.

If he can identify such stable relationships, he may then find it possible to use them as building-blocks, or elements of a computer model. These elements, replicated many times, can be combined and manipulated by the computer (according to rules specified by the model builder) to generate larger, quasi-unique patterns of urban form and process which resemble those of the real world. The model literally consists of "named" variables embedded in mathematical formulae (structural relations), numerical constants (parameters), and a computational method programmed for the computer (algorithm). The pattern generated is typically a set of values for variables of interest to the planner or decision maker, each value tagged by geographic location and/or calendar date of occurrence.

The Uses of Models

The model thus constructed may fall into any of three classes, depending on the interest of the client and the ambition of the model builder. In ascending order of difficulty, these are: descriptive models, predictive models, and planning models.

Descriptions

The builder of a descriptive model has the limited objective of persuading the computer to replicate[3] the relevant features of an existing urban environment or of an already observed process of urban change. Roughly speaking, the measures of his accomplishment are: (1) the ratio of input data required by the model to output data generated by the model; (2) the accuracy and cost of the latter as compared to direct observation of the variables in question; and (3) the applicability of his model to other times and places than that for which it was originally constructed.

Good descriptive models are of scientific value because they reveal much about the structure of the urban environment, reducing the apparent complexity of the observed world to the coherent and rigorous language of mathematical relationships. They provide concrete evidence of the *ways* in which "everything in the city affects everything else," and few planners would fail to benefit from exposure to the inner workings of such models. They may also offer a shortcut to fieldwork, by generating reliable values for hard-to-measure variables from input data consisting of easy-to-measure variables.[4] But they do not directly satisfy the planner's demand for information about the future, or help him to choose among alternative programs. For these purposes, he must look to the more ambitious predictive and planning models.

Predictions

For prediction of the future, an understanding of the relationship between form and process becomes crucial. In a descriptive model it may suffice to note that X and Y are covariant (e.g., that the variable Y consistently has the value of $5X$, or equivalently, that $X = .2Y$); but when the aim is to predict the value of Y at some future time, the model must specify a causal sequence (e.g., that a one-unit change in the value of X will *cause* the value of Y to change by five units). If one is able to postulate the direction of causation, knowledge of the future value of the "cause" enables one to predict the future value of the "effect."[5]

Thus the first task of the builder of a predictive model is to establish a logical framework within which the variables of interest to his client stand at the end rather than at the beginning of a causal sequence. (Variables in this terminal position are often described as "endogenous.") His second task is to make sure that those variables which stand at the beginning (prime causes, often called "exogenous") can be plausibly evaluated as far into the future as may be necessary. These requirements may enlarge his frame of reference far beyond that which would serve for a merely descriptive model.[6]

The second requirement is partly relaxed in the case of *conditional* predictions, which are in any case of greater interest to planners than the unconditional variety. The planner is ordinarily interested in the state of the world following some contemplated act on his part, or following some possible but uncertain event outside his control. The model may then be allowed to respond in the form, "if X occurs, then Y will follow," without explicitly asserting the likelihood of X's occurrence. But explicit predictions must still be made for other exogenous events, since these may reinforce or counteract the effects of the hypothetical change in X.

A special case of conditional prediction is called "impact analysis." Here the interest is focused on the consequence that should be expected to follow a specified exogenous impact (change in X), if the environment were otherwise undisturbed.

PLANNING

Finally, there are planning models, a class whose technology is not far developed. A planning model necessarily incorporates the method of conditional prediction, but it goes further in that outcomes are evaluated in terms of the planner's goals. The essential steps are as follows: (1) specification of alternative programs or actions that might be chosen by the planner; (2) prediction of the consequences of choosing each alternative; (3) scoring these consequences according to a metric of goal achievement; and (4) choosing the alternative which yields the highest score.

The best-known species of planning model executes these steps by means of a "linear program," a computational routine allowing the efficient exploration of a very wide spectrum of alternatives—albeit under rather special restrictions as to permissible cause-effect relationships, and assuming complete information about alternatives and their consequences at the time of choice. Perhaps more relevant to urban planners is the problem of making a sequence of choices, the effects of each choice conditioning the alternatives available for subsequent choices. Since at each decision point there are as many "branches" as alternatives available, the spectrum of possible final outcomes can easily become astronomical. If steps (3) and (4) above are programmed for the computer, it is feasible to trace a fairly large number of alternative decision sequences through to their final outcomes; and mathematicians have reported some success with "dynamic programs" for identifying optimal sequences more efficiently than by trial-and-error.[7]

THEORIES AND MODELS

I have indicated that the model builder's work begins with the identification of persistent relationships among relevant variables, of causal sequences, of a logical framework for the model. In so doing, he must develop or borrow from theories of urban form and process. Although "theory" and "model" are often used interchangeably to denote a logico-mathematical construct of interrelated variables, a distinction can be drawn. In formulating his constructs, the theorist's overriding aims are logical coherence and generality; he is ordinarily content to specify only the conceptual significance of his variables and the general form of their functional interrelationships. The virtuosity of the theorist lies in rigorous logical derivation of interesting and empirically relevant propositions from the most parsimonious set of postulates.

The model builder, on the other hand, is concerned with the application of theories to a concrete case, with the aim of generating empirically relevant output from empirically based input. He is constrained, as the theorist is not, by considerations of cost, of data availability and accuracy, of timeliness, and of the client's convenience. Above all, he is required to be explicit, where the theorist is vague. The exigencies of his trade are such that, even given his high appreciation of "theory," his model is likely to reflect its theoretical origins only in oblique and approximate ways. Mechanisms that "work," however mysteriously, get substituted for those whose virtue lies in theoretical elegance.

The theoretical perspective of the model builder is most clearly visible in the set of structural relations he chooses as the framework of his model. A neatly articulated model will consist of a series of propositions of the general form, $Y = f(U, V, X, Z \ldots)$.[8] These propositions embrace the variables in which he is interested and specify the ways in which these variables act on one another. For most models relating to policy issues, it is useful to classify the propositions in terms of their content as technological, institutional, behavioral, or accounting.[9] While there may well be alternative sets of such propositions that convey the same meaning, the model builder is at least bound by rules of consistency (no contradictory propositions) and coherence (as many independent propositions as there are variables). Within these rules, his choice of structures is guided mostly by his sense of strategic advantage.

The pure theorist is often satisfied with the general forms indicated above, or with these forms plus a few constraints or restrictions. The model builder must be much more explicit, detailing the exact functional forms of his structural relations (e.g., $Y = \log U + a(V/X) - Z^b$); he must also fit his variables (Y, U, V, X, Z) and parameters (a, b) from empirical sources.

The Strategy of Model Design

The "dirty work" of transforming a theory into a model is further discussed below ("Fitting a Model"). At this point I want to review some strategic alternatives of design open to the model builder, choices which demand all his skill and ingenuity since they bear so heavily on the serviceability of his model to its predetermined purposes. Typically, these decisions must be made in an atmosphere of considerable uncertainty with respect to problems of implementation and eventual uses, and there are no clear canons of better and worse. Though the model builder can profit from the experience of others who have dealt with similar problems, he is to a large extent thrown back on his intuitive perceptions and his sense of style.

The Level of Aggregation

Perhaps his most important choice concerns the level of aggregation at which he finds it profitable to search for regularities of form and process. While there is an accepted distinction between macroanalysis and microanalysis, the differences between these modes of perception can be elusive. Neither is the exclusive property of a particular academic discipline, but in urban studies, macroanalysis is closely associated with urban geography, demography, social physics, and human ecology, while microanalysis is typically the métier of economics and social psychology.

The geographers, demographers, ecologists, and social physicists prefer to deal with statistics of mass behavior and the properties of collectivities. The elements of a model based on this tradition are likely to be stock-flow parameters, gravity or potential functions, matrices of transition probabilities.[10] Faced with the same *explanandum*, the economist is much more likely to think in terms of a market model, in which resources are allocated or events determined through competitive interaction of optimizing individuals whose behavior is predicated on a theory of rational choice. The social psychologist also works from a theory of individual choice, and has his own version of the market model—though it is less articulate because it embraces a much wider variety of transactions.[11]

The principal criticism of the macroanalytic approach is that its "theory" consists in large part of descriptive generalizations which lack explicit causal structure. Thus a macromodel of residential mobility may consist essentially of a set of mobility rates for population subgroups classified by age, sex, or family status, rates based on historical evidence of the statistical frequency of movement by the members of such groups. For purposes of prediction, one may assume that these rates will apply to future as well as past populations; but since the reasons people move are not explicit in such a model, the assumption of continuity in behavior cannot be easily modified to fit probable or postulated future changes in the environment of this behavior.

A second objection to macroanalytic approaches is that they do not lend themselves easily to financial accounting schemes. These are of particular relevance to planning models whose purpose is to distinguish among better and worse alternatives of policy or program. Strictly speaking, such distinctions can only be made if goal achievement is reducible to a single metric, and the most comprehensive metric available in our society, whether we like it or not, is money.[12] Thus in choosing among alternative transportation plans, the objective may be to maximize net social return to transportation investments—for example, to maximize the difference between benefits to be derived from the investment and costs allocable to it. Even though a gravity-model representation of the journey-to-work/residential location relationship may "work" in the sense of generating accurate predictions of population distribution and travel patterns, it will not yield financial data so easily as a market model of travel behavior and residential site selection, since the latter operates throughout in terms of price-defined alternatives faced by households.

The microanalytic approach also has its problems. Chief among these is that a model based on the theory of rational choice can be implemented only if the chooser's system of relative values—technically, his "preference system"—can be specified in considerable detail. The search for an empirical technique to achieve this detailed specification has frustrated generations of economists, and approximations to date are both crude in detail and based on highly questionable operating assumptions. Lacking the ability to observe these preference systems directly, the modeler is restricted to a very meager menu of empirically relevant propositions concerning the complementarity and substitutability of economic goods, propositions deducible from general theoretical principles.

The second problem of the microanalytic approach is the implementation of a comprehensive market model—one embracing the entire range of transactions which substantially affect the patterns of urban development and land use. Given complete information about the demand schedules of buyers and the supply schedules of sellers, the classical theory of a perfectly competitive market for a homogeneous commodity is simple enough, having a determinate solution for both the volume of transactions and the emergent price of the commodity. But the model builder is faced, empirically, with a congeries of interrelated markets, subtly-differentiated commodities, imperfections in communication, and inequities of bargaining position, all of which rule out the easy mathematical resolutions of the classical case. The fact is that we are presently able to implement only quite crude and tenuous approximations of market models.[13]

THE TREATMENT OF TIME

Except for the simple descriptive case, a model usually purports to represent the outcome of a process with temporal dimensions. Beginning with the state of the (relevant) world at time t, it carries us forward to the state of that world at $t + n$; thus a land-use model may start with a 1960 land-use inventory in order to predict the 1970 inventory. The way in which this time dimension is conceived is a matter of considerable strategic significance; the choice lies among varying degrees of temporal continuity, ranging from comparative statics at one extreme, through various types of recursive progression, to analytical dynamics at the other extreme.

At first glance, the choice seems to hinge merely on the question, how often need results be read out? But the issues go deeper, involving the model builder's perception of the self-equilibrating features of the world represented by his model, the empirical evaluation of response lags among his variables, and his interest in impact analysis as distinguished from other types of conditional or unconditional prediction.

The method of comparative statics implies a conviction that the system is strongly self-equilibrating, that the endogenous variables respond quickly and fully to exogenous changes. The model's parameters, fit from cross-section data, represent "equilibrium" relationships between exogenous and endogenous variables; a prediction requires specification of the values of the exogenous variables as of the target date. The process by which the system moves from its initial to its terminal state is unspecified.[14]

Alternatively, comparative statics may be used for impact analysis, where no target date is specified. Assuming only one or a few exogenous changes, the model is solved to indicate the characteristics of the equilibrium state toward which the system would tend in the absence of further exogenous impacts.

Self-equilibration is not a necessary assumption for analytical dynamics, an approach which focuses attention on the processes of change rather than on the emergent state of the system at a specified future date. Technically, this type of model must be formulated as a set of differential equations, at least some of which include variables whose rates of change are specified with respect to time.

Implementation of such a model requires only specification of its structural parameters and the "initial conditions" of its variables.

Thereafter, all processes are endogenous except time, and the time path of any variable can be continuously traced. The state of the system can be evaluated at any point in time. If the system *is* self-equilibrating, the values of its variables should converge on those indicated by analogous comparative statics; but without self-equilibrating properties, the system may fluctuate cyclically, explode, or degenerate.

Because comparative statics requires such strong equilibrium assumptions (seldom warranted for models of urban phenomena), and because analytical dynamics requires virtually complete closure (all variables except time are endogenous), most model builders compromise on recursive progressions. This method protrays the system's changes over time in lock-step fashion by means of lagged variables, for example:

(1) $$Y_{t+1} = a + b\,X_t$$
(2) $$X_t = c + d\,Y_{t-1}$$

Starting with initial values for either X or Y, one carries the system forward by alternately solving Eqs. 1 and 2. Of course in this example, a bit of algebraic manipulation suffices to evaluate Y_{t+n} directly from a given Y_t; but the case is seldom so simple—and the model builder is likely to want to inject periodic exogenous changes into this recursive sequence.

THE CONCEPT OF CHANGE

Any model dealing with changes over time in an urban system must distinguish (at least implicitly) between variables conceived as "stocks" and variables conceived as "flows." A stock is an inventory of items sufficiently alike to be treated as having only the dimension of size or number—for example, dwelling units, female labor-force participants, acres of space used for retail trade. This inventory may change as items are added or deleted; such changes, expressed per unit of time, are called flows. A model builder may choose to focus either on the factors which determine the magnitude of each stock, or on the factors which determine the magnitude of each flow.[15]

Since a stock is by definition the integral over time of the corresponding flow, it must also have the same determinants as the flow. But if the model builder limits his attention to flows which occur over any short span of time, he can afford to take a number of shortcuts. Exogenous variables whose effects on stocks are visible only in the long run can be ignored or treated as fixed parameters. Whereas nonlinear expressions may be necessary to represent the long-run growth of a stock, marginal increments in the short run can often be represented by linear expressions. By accepting the initial magnitude of a stock as historically "given," one avoids the necessity of replicating the past and can devote himself to modeling the events of the present and near future.

Consider a model of retail location whose eventual application will be a five-year projection of the distribution of retail establishments within an urban area. The existing pattern (initial stock) of retail establishments in a large city reflects locational decisions made over the course of a century or more, during which time the transportation system, merchandising techniques, and patterns of consumption all have changed slowly but cumulatively. Most of the present stock of retail establishments will still be in operation at their present sites five years hence.

If the model builder is willing to organize his design around the *present* characteristics of the transportation network, of merchandising methods, of consumption patterns, his task may be greatly simplified. And the resulting model may be quite adequate for the prediction of short-run *changes* in retail location (say, as a consequence of population growth), even though it would not be able to recapitulate the city's history of retail development.

Clearly the model builder must weight the advantages of such simplifications against the fact that his model will have a shorter useful life. Since its structure postulates stability in a changing environment, the model will soon lose its empirical relevance.[16] By way of compromise, many model builders make use of "drift parameters": structural "constants" which are programmed for periodic revision to reflect changing environmental conditions, conditions which cannot conveniently be made explicit in the model.

SOLUTION METHODS

An integral part of the strategy of model design is a plan for operating the model—an algorithm or method of solution. This plan describes the concrete steps to be taken from the time that input data is fed to the computer until final results are read out. Four general methods are prominent; the choice among them is largely governed by the degree of logico-mathematical coherence of the model itself.

The neatest and most elegant method is the analytic solution. Ordinarily this method is applicable only to models which exhibit very tight logical structures and whose internal functional relationships are uncomplicated by nonlinearities and discontinuities. In substance, the set of equations constituting the model is resolved by analysis into a direct relationship between the relevant output variables and the set of input variables; intervening variables drop out of the "reduced form" equations. The paradigm system used above to illustrate recursion (Eqs. 1 and 2) can be solved analytically; for example:[17]

$$(3) \quad Y_{t+4} = (a + bc)\,(1 + bd) + (bd)^2 Y_t$$

For models lacking complete logical closure, or whose structures are overburdened with inconvenient mathematical relationships, an alternative to the analytic solution is the iterative method. This method comprises a search for a set of output values which satisfy all the equations of the model; it proceeds initially by assuming approximate values for some of the variables and solving analytically for the remainder. These first-round solutions are then used as the basis for computing second approximations to replace the initially estimated values, and so on. Except for various degenerate cases, the solution values eventually "converge"—that is, further iterations fail to result in significant changes in the solution. Mechanically, the process is quite similar to recursive progression of a self-equilibrating system, but the iterative process need not imply either a sequence over time or a causal sequence. A drawback of this method is that it fails to signal the existence of alternative solution sets; a possibility that may have considerable importance for the interpretation of results.[18]

Ambitious models of urban processes may not meet the requirements for either the analytic or iterative methods of solution because of their scope: In the attempt to embrace a wide range of obviously relevant phenomena, one easily loses mathematical rigor and logical closure. For models of this class—loosely articulated "system analyses"—machine simulation may be the best resort. The model specifies an inventory of possible "events" and indicates the immediate consequences of each event for one or more variables representing a "stock" or population. A change in the magnitude of a stock has specified (endogenous) consequences in the form of inducing new events; but characteristically, the major source of new events is exogenous. Indeed, the more sophisticated simulations (Monte Carlo or stochastic models) generate exogenous events by random choice from a given frequency distribution of possibilities. The computer's principal task is to keep a running account of all stocks and to alter them in response to events. This method is less appropriate for explicit projections than for tests of the sensitivity of the model (and by implication, of the real-world system represented by the model) to various possible constellations of exogenous events.

Finally, there is the method of "man-machine simulation," in which computer processing of input data is periodically interrupted, and the intermediate state of the system is read out for examination by a human participant. He may adjust intermediate results to correspond with his judgment as to their inherent plausibility, or he may use these intermediate results as a basis for a "policy" decision which is then fed back to the computer model as an exogenous change in values for specified variables or parameters. The human participant is ordinarily included for educational reasons—to give him practice in responding to planning problems—but on occasion, he is there simply because the model builder does not fully trust his model to behave "sensibly" under unusual circumstances.[19] (See below, "Parameters.")

FITTING A MODEL

Once the model builder has selected a theoretical perspective, designed a logical framework large enough to encompass his objectives, and postulated the existence of enough empirical regularities to permit the resolution of his problem, his next task is to "fit" or "calibrate" the model. This task involves two types of transformation: the variables mentioned in the model must be given precise empirical definition, and numerical values must be provided for the model's parameters.

Variables

The first transformation always involves compromise. A variable conceived in general terms (household income) must be related to an available statistic (median income of families and unrelated individuals as reported by the U. S. Census of 1960 on the basis of a 25 per cent sample), and the restrictions and qualifications surrounding the data must be carefully explored to be sure they do not seriously undermine the proposed role of the variable in the model (aggregation of medians is difficult; response errors may create serious biases in the data; sampling variability of figures reported for small areas may be uncomfortably large).

A variable included in the model because of its theoretical significance may not be directly observable in the real world, so that some more accessible proxy must be chosen. Thus many land-use models deal in "location rents" (defined as that portion of the annual payment to an owner of a parcel of land which is attributable to the geographical position of the parcel as distinct from its soil or slope characteristics, existing structural improvements, or services provided by the landlord), but empirical sources tell us only about "contract rents" (the total contractual payment of tenant to landlord). Can contract rents be statistically standardized to serve as a reliable proxy for location rents?

I know of no formal canon of method for fitting variables, although I can think of some scattered principles to be observed.[20] More frequently than not, the problems encountered at this step force the model builder to backtrack and revise parts of his logical structure to lessen its sensitivity to bad data or to make better use of what data is actually available. Since few published statistics are exactly what they seem to be from the table headings and column stubs, it is very easy for one inexperienced in the generation of a particular class of data to misinterpret either its meaning or its reliability.

Parameters

The fitting of parameters—numerical constants of relationship—is necessary for two reasons: (1) theoretical principles and deductive reasoning therefrom are seldom sufficient to indicate more than the appropriate sign (positive or negative) and probable order of magnitude for such constants; and (2) since these constants are measures of relationship between numerical variables, the precise empirical definition of the variable affects the value of the parameter. For instance, the appropriate value of a labor-force participation rate depends among other things on whether the pool from which participants are drawn is defined to include persons 15 to 60 or persons 14 to 65.

Parameter fitting is a highly developed branch of statistical method.[21] The most common tool is regression analysis, the simplest case being the estimation of parameters for a linear function of two variables, $Y = a + bX$. From a set of coordinate observations of the values of X and Y, one can estimate values for a and b in such a way as to minimize the expected error of estimate of Y from known values of X.

If the model can be formulated as a set of simultaneous linear equations, an elaboration of this method can be used to locate "best fit" values for all parameters in the system.[22] Models fitted in this way are often described as "econometric," although the method is equally applicable to noneconomic variables. A significant drawback of econometric fitting is that the criterion of selection for the values assigned to each parameter is the best *overall* fit of the model to a given array of data. The values generated for individual parameters are often surprising, yet it is difficult to look "inside" the fitting process for clues of explanation.

Alternatives to a comprehensive econometric fit can be described generally as "heuristic" methods. The model is partitioned into smaller systems of equations—some perhaps containing a single parameter—so that the parameters of each subsystem can be fit independently. This is in fact the typical approach, since few large models of urban form and process can be formulated as a single system of linear equations and still meet the objectives of the client.

Methods for obtaining estimates of the various parameters in these subsystems may vary considerably. A model ordinarily contains parameters whose function is nominal, and a model builder anxious to get on with his job may simply assign an arbitrary but plausible value to such a parameter. Where the context rules out direct methods for deriving simultaneous "best fits" even for the parameters of a limited subsystem, trial-and-error methods can be used to find a set of parametric values which seem to work. Or parametric values may be taken directly from empirical analogs, without regard for "best fit" in the context of the model.[23]

Finally, I should mention that model builders sometimes despair of finding a mathematically exact expression of relationship among certain of their model variables, so resort instead to "human" parameters. At the appropriate point in the operation of the model, intermediate or preliminary results are scanned by persons of respected judgment, who are asked to alter these outputs to conform to an intuitive standard of plausibility based on their experience in the field. The altered data are then fed back to the computer for further processing.

Testing a Model

Fitting a model is analogous to the manufacture and assembly of a new piece of electrical machinery. A work team, guided by engineering drawings, fabricates each component and installs it in proper relation to other components, connecting input-output terminals. Along the way, considerable redesign, tinkering, and mutual adjustment of parts is inevitable; but eventually the prototype is completed. However carefully the individual components have been tested, and their interconnections inspected, a question remains about the final product: Will it really work?

Industrial experience indicates that the best way to answer this question is to turn the machine on and apply it to the task at hand. This precept applies also to computer models of urban form and process, with the important reservation that it is extremely difficult to select a "fair" but revelatory task, or to establish clear and objective standards of performance.

The appropriate test for a model depends, of course, on its predetermined function. It is unfair to ask a descriptive model to make a prediction, or a predictive model to find the optimal solution to a planning problem. But it is unfortunately the case that even an appropriate test may be infeasible.

The easiest model to test is the descriptive variety. Thus, for a model of urban form, the appropriate test would be its ability to replicate the details of an existing urban pattern on the basis of limited information concerning the area in question. Since most such models are built with a particular urban area in mind, and fitted with reference to this area's characteristics one ordinarily has detailed observations (for example, concurrent and otherwise compatible inventories of land use, structures, human populations, business enterprises, transportation facilities, and so forth) against which the model's output may be checked. The limitations of the test should also be apparent: The model's structure and parameters may be so closely locked into the patterns evident in this particular area and time that its descriptive abilities may have no generality; applied to another city the model may fail miserably.

The appropriate test for a predictive model is to run a prediction and verify the details of its outcome. The more distant the horizon of forecast, the more stringent the test; it would be easy to predict the distribution of workplaces in Boston tomorrow if one were given today's inventory. But few clients have the patience to finance several years of model building, then wait several more years to verify the model's first predictions. And even if one were willing to wait, there is the further problem that the model will almost certainly be designed for *conditional* predictions, and it would be remarkable indeed to discover in retrospect that all postulated conditions had been fulfilled.

The more accessible alternative is *ex post facto* prediction: Take the state of the world in 1950 as a starting point and apply the model by forecasting for 1960, then compare the forecast values to the observed values for 1960.[24] This procedure is likely to suffer from the same limitation of semicircularity that plagues the testing of descriptive models. More likely than not, the predictive model was *fitted* to the recorded processes of change, 1950–60. And if not, the reason is likely to be that comparable data are not available for the two dates. A predictive model is oriented to the problems of the future, and the model builder is anxious to feed his model the most recent additions to the menu of urban data—indeed, he may well initiate field work on a new series to provide it with a balanced diet. Why limit his freedom by insisting that his model be able to subsist on the more limited menu available a decade ago?

The test of a planning model has two distinct phases. The first is a check on its ability to trace through the consequences of a given planning decision or set of decisions; this phase is a form of conditional prediction, and subject to all the hazards described above. The second phase is a check on the ability of the model to select an optimal result from a spectrum of alternative outcomes. It may fail to do so because (1) short-cut methods may eliminate as suboptimal some outcomes which have more promise than they immediately show; (2) the evaluation of outcomes may be very sensitive to engineering estimates of cost or imputation of benefits, and these are intrinsically nebulous; or (3) the criteria of selection may be poorly stated, so that an outcome which would in fact be acceptable to the client is classified as unacceptable by the model.

"Sensitivity testing" is sometimes urged as a more accessible substitute for the performance tests discussed above; although it is easy to perform and applicable to a wide variety of models, sensitivity testing elicits indications of the "strength" of a model's design rather than of its descriptive or predictive or evaluative accuracy. The procedure is as follows: By varying the value of a single parameter (or even of an input variable) in successive runs of the model, one can measure the difference in outcome associated with a given parametric change. If the model's response to wide differences in parametric values is insignificant, this may be an indication that the parameter—and the associated network of functional relations—is superfluous. On the other hand, extreme sensitivity of outcomes to parametric changes indicates either that the parameter in question had better be fit with great care, or that some further elaboration of this component of the model is in order—on the grounds that the analogous real-world system must in fact have built-in compensations to forestall wild fluctuations in outcome.

EVALUATION

The picture I have painted above is rather grim, but I think it is accurate. The truth is that the client ordinarily accepts from the model builder a tool of unknown efficacy. The tests that the client can reasonably insist upon are at best partial and indecisive. Perhaps worst of all, those who must make the major decisions about sponsoring a model-building project are unlikely to have the time or training to evaluate a proposal, and later, having footed a large bill, have a vested interest in the model hardly second to the professional stake of its builder. In the absence of incontrovertible evidence to the contrary, the builder and sponsor will agree that the model "works."

In the face of such ambiguities, it is not hard to imagine a reasonable man's refusal to participate in such a probable boondoggle. But for the reasons indicated at the beginning of this article, I do not anticipate any shortage of sponsorship for model-building projects: It is better to try something—anything—than to merely wring one's hands over the futility of it all. Sponsors and model builders too can take comfort in the thought that they are building for the distant if not the near future.

Above all, the process of model building is educational. The participants invariably find their perceptions sharpened, their horizons expanded, their professional skills augmented. The mere necessity of framing questions carefully does much to dispel the fog of slopply thinking that surrounds our efforts at civic betterment. My parting advice to the planning profession is: If you do sponsor a model, be sure your staff is deeply involved in its design and calibration. The most valuable function of the model will be lost if it is treated by the planners as a magic box which yields answers at the touch of a button.

FOOTNOTES

[1] Wingo in W. Z. Hirsch, ed., *Elements of Regional Accounts* (Baltimore: Johns Hopkins Press, 1964), p. 144. Model building has also been greatly encouraged by the electronic revolution in data processing and computation; mathematical models have an insatiable appetite for numbers.

[2] An immensely important topic in the field of model design which is not covered by this article is the joint effort of model builder and client to define the "problem" to which a model offers a possible "solution." On this point, I know of no better reference than a RAND book on systems analysis, E. S. Quade, ed., *Analysis for Military Decisions*. R-387-PR (Santa Monica: The RAND Corporation, 1964), particularly the essays of R. D. Specht ("The Why and How of Model-Building"), R. McKean ("Criteria"), and E. S. Quade ("Pitfalls in Systems Analysis").

[3] Some model builders would freely substitute "simulate" for my "replicate." All models are intended in some sense to simulate reality, but this usage is the source of some confusion in the literature since "simulation" has acquired another, more technical meaning descriptive of a class of algorithms. In this essay, I use the term only in the latter sense. See, below, "Solution Methods."

[4] For example, traffic analysts use zonal interchange models to generate estimates of zone-to-zone traffic flows from inventories of the land uses in each zone. One prominent member of the profession is so convinced of the descriptive reliability of these models that he sees no further need for direct surveys of traffic movements (O & D studies).

[5] Philosophers of science view the concepts of "cause" and "effect" with jaundiced eyes. For lesser mortals these concepts are most helpful, and not at all dangerous so long as they are applied within the framework of a system of interdependence. H. A. Simon, *Models of Man* (New York: John Wiley & Sons, Inc., 1957), Chaps. 1–3.

[6] No variable is intrinsically endogenous or exogenous. These terms, like the statistician's "dependent" and "independent," merely define the position of a variable within a particular model. A further useful distinction can be made between exogenous variables subject to policy control and those which are not, and between endogenous variables of direct interest to the planner and those which are included only because they are necessary to complete the logical structure of the model. S. Sonenblum, and L. H. Stern. "The Use of Economic Projections in Planning," *Journal of the American Institute of Planners*, **30** (1964), 110–123.

[7] The fundamentals and applications of linear programming are summarized in very readable form by W. J. Baumol, "Activity Analysis in One Lesson," *American Economic Review*, **48** (1958), 837–873. I cannot find any simple exposition of dynamic programming, but see R. Bellman, *Dynamic Programming* (Princeton: Princeton University Press, 1957) for a brief account of the class of problems to which the technique is applicable.

[8] "The value of Y is a function of (depends on) the values of U, V, X, and Z, and so forth." For a gentle introduction to the notation and methods of mathematical modeling, Beach is an excellent source. E. F. Beach, *Economic Models: An Exposition* (New York: John Wiley & Sons, Inc., 1957).

[9] Some examples, in prose rather than symbols:

Technological: The maximum vehicular capacity of a roadway is a function of the number of lanes, the average distance between signals, and the weather.

Institutional: Disposable family income is a function of gross family earnings and the tax rate.

Behavioral: The level of housing density chosen by a family depends on disposable family income, the average age of family members, and the location of the work place of the principal wage earner.

Accounting: Total land in use is the sum of land in residential use, in retail use, in manufacturing use, and so forth.

[10] The essays in Part II of G. K. Zipf, *Human Behavior and the Principle of Least Effort* (Reading, Mass.: Addison-Wesley Publishing Company, Inc., 1949) should give the reader a "feel" for the macroanalytic perspective in urban models. See also G. A. P. Carrothers, "An Historical Review of the Gravity and Potential Concepts of Human Interaction," *Journal of the American Institute of Planners*, **22** (1956), 94–102 and B. J. L. Berry, "Cities as Systems Within Systems of Cities." Paper presented at the Annual Meeting of the Regional Science Association, Chicago (Nov. 1963).

[11] Dyckman provides an excellent review of the theory of rational choice in a planning context. J. W. Dyckman, "Planning and Decision Theory," *Journal of the American Institute of Planners*, **27** (1961), 335–345. Any introductory text in economics will describe the microanalytic underpinnings of demand and supply schedules and will also review a family of market models. The most ambitious microanalytic model ever undertaken in the social sciences is described in G. H. Orcutt, M. Greenberger, J. Korbel, and A. M. Rivlin, *Microanalysis of Socioeconomic Systems: A Simulation Study* (New York: Harper & Row, Publishers, 1961). For models that embrace more than "economic" man, see Simon, *op. cit.* or P. F. Lazarsfeld, ed., *Mathematical Thinking in the Social Sciences* (New York: Free Press of Glencoe, Inc., 1954).

[12] N. Lichfield, "Cost-Benefit Analysis in City Planning," *Journal of the American Institute of Planners*, **26** (1960), 273–279.

[13] It is my personal conviction—not shared by all members of the fraternity of model builders—that the macroanalytic approach to modeling urban form and processes shows the greater promise of providing reliable answers to concrete problems of prediction and planning. For a contrary view, see the forceful statement by B. Harris, "Some Problems in the Theory of Intra-Urban Location." Paper prepared for a seminar sponsored by the Commitee on Urban Economics of Resources for the Future, Washington, D. C. (Apr. 1961).

[14] Descriptive models of urban form are nearly always static or "equilibrium" models, and are sometimes used for quasi-predictions (comparsative tatics). For convenient examples, see B. Harris, "A Model of Locational Equilibrium for Retail Trade." Paper prepared for the Seminar on Models of Land Use Development, Institute of Urban Studies, University of Pennsylvania, Philadelphia (Oct. 1964) or I. S. Lowry, *A Model of Metropolis*. RM-4035-RC (Santa Monica: The RAND Corporation, 1964).

[15] Contrast the emphasis on stocks in the San Francisco CRP model designed by Arthur D. Little, Inc., "A Simulation Model of the Residential Space Market in San Francisco." Paper prepared for the Seminar on Models of Land Use Development, Institute

of Urban Studies, University of Pennsylvania, Philadelphia (Oct. 1964), with the emphasis on flows in R. S. Bolan, W. B. Hansen, N. A. Irwin, and K. H. Dieter, "Planning Applications of a Simulation Model." Paper prepared for the New England Section, Regional Science Association, Fall Meeting, Boston College (Oct. 1963) or with the several "growth allocation" models described in this issue of the *Journal*.

[16] R. V. Black, "Scientific Versus Empirical Projections," *Journal of the American Institute of Planners*, **26** (1960), 144–145.

[17] The reader is warned that Eq. 3 is not a general solution for any Y_{t+n}, but merely the simplest expression for Y_{t+4}.

[18] An example of the iterative technique is given in some detail in Lowry, *op. cit.*, pp. 12–19.

[19] A good bibliography in simulation methods is M. Shubik, "Bibliography on Simulation, Gaming, Artificial Intelligence and Allied Topics," *Journal of the American Statistical Association*, **55** (1960), 736–751. M. A. Geisler, W. W. Haythorn, and W. A. Steger, *Simulation and the Logistics Systems Laboratory*. RM-3281-PR (Santa Monica: The RAND Corporation, 1962) offer a quick and readable review of the field, with emphasis on man-machine simulation, or "gaming." N. D. Grundstein, "Computer Simulation of a Community for Gaming." Paper prepared for the Annual Meeting of the American Association for the Advancement of Science, Denver (Dec. 1961), describes a "community game" for the training of planners and municipal administrators.

[20] Special data problems encountered in modeling urban form and process are discussed by B. Harris, "An Accounts Framework for Metropolitan Models," in Hirrch, *op. cit.*, 107–127. Also See W. A. Steger, "Data and Information Management in a Large Scale Modelling Effort: The Pittsburgh Urban Renewal Simulation Model." Paper prepared for the Seminar on Models of Land Use Development, Institute of Urban Studies, University of Pennsylvania, Philadelphia (Oct. 1964), 1–6.

[21] Beach, *op. cit.* Part II, provides an especially good introduction to statistical and econometric methods.

[22] The convenience of this method is so great that it is often applied to systems containing known nonlinearities, on the grounds that a linear approximation is better than nothing. Simultaneous estimation of the parameters of nonlinear systems is possible, but more difficult; the outstanding example among land-use models is Karl Dieter's Program Polimetric for fitting an exponential model with a great many parameters. (The model, but not the fitting method, is described in R. S. Bolan, *op. cit.*

[23] J. H. Niedercorn, *An Econometric Model of Metropolitan Employment and Population Growth*. RM-3758-RC (Santa Monica: The RAND Corporation, 1963). His model is partitioned into three subsystems, each of which was fit independently. The discussion on pp. 14–15 illustrates the variety of estimating methods ordinarily required to fit a model. See also Harris, *op. cit.* for a discussion of the "gradient search" method of estimating parameters.

[24] D. M. Hill, "A Growth Allocation Model for the Boston Region—Its Development, Calibration and Validation." Paper prepared for the Seminar on Models of Land Use Development, Institute of Urban Studies, University of Pennsylvania, Philadelphia (Oct. 1964), reports with unusual thoroughness on a test of this type for the EMPIRIC Model developed for Boston by Traffic Research Corporation.