A Theory of Urban Growth

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In an economy experiencing endogenous economic growth and exogenous population growth, we explore two main themes: how urbanization affects efficiency of the growth process and how growth affects patterns of urbanization. Localized information spillovers promote agglomeration and human capital accumulation fosters endogenous growth. Individual city sizes grow with local human capital accumulation and knowledge spillovers; and city numbers generally increase, which we demonstrate is consistent with empirical evidence. We analyze whether local governments can successfully internalize local dynamic externalities. In addition, we explore how growth involves real income differences across city types and how urbanization can foster income inequality.

Most nonagricultural production in developed countries occurs in metropolitan areas. The underlying reasons why economic activity agglomerates into cities—localized information and knowledge spillover—also make cities the engines of economic growth in an

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economy (Lucas 1988). Consequently, urbanization strongly influences the growth process, influencing both the efficiency of growth and the extent of income inequality within an economy. In turn, growth influences the urbanization process, driving the spatial evolution of production and population agglomeration. This paper models an economy composed of cities of endogenous size and number, under conditions of exogenous population growth and endogenous economic growth, exploring two main themes: how urbanization affects efficiency of the growth process and how growth affects the nature and patterns of urbanization. We comment also on the role of urbanization in determining income inequality.

The first theme derives from key features common to both the endogenous growth and urban literatures: external scale economies and knowledge spillovers. External scale economies (Romer 1986) and knowledge spillovers (Lucas 1988), which augment returns to private human capital accumulation, drive long-run endogenous growth. Scale economies are a key feature in the urban literature, starting with Marshall's (1890) frequently cited passages on how cities provide an environment of close contacts and neighborhoods generating localized information spillovers. Recent work has developed microfoundations for scale economies such as information spillovers (Fujita and Ogawa 1982) and search and matching in local labor markets (Helsley and Strange 1990) and has quantified their magnitude (e.g., Sveikauskas 1975; Nakamura 1985; Henderson 1986; Ciccone and Hall 1996). An important theoretical result from the literature is that either autonomous, competitive local governments or competitive large-scale land developers can internalize localized information externalities, leading to an efficient national allocation of resources in a static context (Flatters, Henderson, and Mieszkowski 1974; Stiglitz 1977; Henderson 1988). Of particular interest in this paper is whether, also, local knowledge spillover externalities can be internalized successfully by such agents in a dynamic context with knowledge accumulation.

The role of knowledge spillovers has been a focus of recent empirical work showing that local average human capital levels affect individual earnings (Rauch 1993*b*) and that knowledge spillovers as evidenced in spatial patterns of patent citations are strongly localized (Jaffe, Trajtenberg, and Henderson 1993). While some attempt has been made to model localized knowledge spillovers in an urban context (Eaton and Eckstein 1997), the role of autonomous local governments or large-scale land developers in enhancing the efficiency of the rate of knowledge accumulation remains an unexplored topic. One of the goals of this paper is to explore under what institutional arrangements, if any, autonomous local authorities can foster more efficient investment in knowledge, an important issue in thinking about local public investment in schooling.

The second theme of the paper is that economic growth affects urban growth and spatial evolution. We develop a model of growth in an urbanized economy that is consistent with basic patterns of urban form and evolution. Black and Henderson (1997) explore such patterns for the United States, mostly for the 1900–1950 time period of rapid urbanization and considerable national turbulence (the Great Depression and two world wars). Three patterns emerge from this and related studies. The first involves growth in the number and sizes of cities. During the 1900–1950 time period, average metropolitan area populations tripled and the number of metro areas doubled. Despite this growth in individual city sizes, in every decade the number of cities increased, fueled by national urbanization (increasing the national percentage urbanized from about 40 to 60 percent) and national population growth (averaging 1.4 percent per year). It is tempting to correlate the growth in city sizes with the tremendous increases in average human capital during this time period, when, for example, the percentage of the 17-year-old population that had completed high school rose from 6.3 to 57.4 percent nationally. We shall pursue this notion in this paper.

The second pattern that emerges concerns the evolution of the relative size distribution of metropolitan areas from decade to decade. Black and Henderson (1997) model transitions as a stationary first-order Markov process. Despite entry of new metropolitan areas, the relative size distribution of cities is astonishingly stable over time, exhibiting no tendency to collapse (‘‘converge’’ to a common city size), spread, go bimodal, and so forth, with the actual distribution fluctuating little between decades. Moreover, stationarity of the transition process itself from decade to decade cannot be rejected.

The third pattern that is examined in Black and Henderson (1997), unlike the previous two, characterizes the static form of an economy rather than its evolution. But it indicates why economies support a wide size distribution of cities that can remain stable over time. We test for the existence of production specialization across cities, using cluster analysis to allocate 317 metro areas to 55 city types, on the basis of 1992 metro area private employment broken into 80 two-digit industries. We also examine how city sizes and human capital levels vary across city types. As F-tests confirm, production patterns indeed differ very significantly across these city types and city sizes and educational attainments are related to city type. For example, cities specializing in financial, business, or diversified services (education, management, engineering, and some business) are significantly larger than traditional manufacturing cities. Electronics, health services, and computer manufacturing type cities have much greater per capita educational attainment than food processing, primary metals, furniture, and textile type cities.

In summary, with respect to the second theme, a goal of the paper is to develop a model of urban evolution that is consistent with basic observed patterns. An urbanized economy has different types of cities specialized in different traded goods, with city sizes and educational attainments varying by city type. This implies that at a point in time there is a nondegenerate relative size distribution of cities. With economic and population growth, city sizes and numbers, respectively, grow over time. Growth, which is parallel across city types, will maintain a stable relative size distribution. One key theoretical and empirical result we establish is the relationship between individual city population growth rates and local human capital growth rates.

Finally, in the paper we shall note that growth under urbanization may foster real income inequality. From the work of Benabou (1993) and Durlauf (1996), we know that localized peer group effects and parental choices of neighborhoods and human capital investments lead to geographic stratification of the population into neighborhoods, which can result in persistent inequalities in real and nominal incomes over time. Though we do not focus primarily on inequality in this paper, we do explore some issues that arise when considering the effect of systematic differences among cities on income inequality, extending the focus of previous studies on inequality arising from neighborhood stratification within cities. Specialization in production patterns across cities occurs in models with scale externalities (Henderson 1974) or in spatial hierarchy models of cities in an agricultural setting (Fujita, Krugman, and Mora 1995). Different types of specialized cities will have different sizes and different equilibrium and efficient per capita levels of human capital, given by the private and social returns to local investment in human capital in each type of city. This will lead to measured real and nominal income inequality across city types. We shall note how under some specifications of the mechanism through which human capital is transmitted across generations, urbanization can lead to evolving true inequality among initially identical dynasties.

In terms of the literature, apart from older, exogenous urban growth models (see Henderson [1988] for a review), this paper is most closely related to the paper by Eaton and Eckstein (1997), who consider human capital spillovers both within and across cities (the latter is an exciting extension). However, in Eaton and Eckstein, the number of cities is fixed and the endogenous growth framework is incompletely specified. In this paper, we step back and for the first time solve a fully specified urban growth model, focusing on issues of city formation and the effect of endogenous growth on changes in city sizes, numbers, and human capital levels over time. Potential links between urbanization and inequality relate this work to Benabou (1993, 1996) and Durlauf (1996), although these papers are not concerned with national spatial evolution per se and do not allow for city formation and growth and multiple numbers and types of cities. And, as noted earlier, the issue of growth efficiency as it relates to urbanization and urban institutions is a topic not yet explored in the literature.

I. A Growth Model of an Urbanized Economy

The growth model of an urbanized economy consists of two components. First is the urban part, which describes the spatial organization of production and population. In an economy, there is a city formation process in national land markets, involving either land developers or autonomous local governments. We assume that the economy consists of only two types of cities, each performing different functions and having different equilibrium sizes and human capital levels and incomes per worker. There are many cities of each type. While having two types of cities provides for a limited city size distribution, it is sufficient to establish basic principles. Type 1 cities in the economy produce the numeraire good, an intermediate input (e.g., materials or disposable machines), that is purchased by firms in type 2 cities. Firms in type 2 cities specialize in production of the economy's consumption good, priced at P relative to the numeraire.

This characterization of cities as being absolutely specialized in traded good production with no costs of intercity trade begs questions about the role of more diversified mega-cities in an economy. For example, Glaeser et al. (1992) suggest that diversification contributes to growth. As in note 2 below, we could have a Dixit-Stiglitz (1977) type basis for urban scale economies based on diversification in local nontraded intermediate input sectors in cities. But even if local diversification is important, we still expect specialization in broader classes of traded goods across cities. Even with enormous diversification, New York's industrial composition with a relative focus on finance, publishing, and fashion looks very different from that of Gary, Indiana. Our stark simplification serves to establish basic principles about growth.

The second component to an urban growth model involves family migration and human capital investment decisions. Workers are members of dynastic families. At the end of the paper, we shall show that our basic results—allocations of people and human capital across cities—hold in a simple overlapping generations model. For dynasties, each family starts with the same human capital per person and each family's size grows at the same rate, g. Each family discounts the future at a rate ρ, where $\rho > g$ to help ensure well-behaved solutions. At each instant, dynasties choose how much total family income to allocate to consumption per member, c, and how much to allocate to increasing the family's human capital stock. Families must allocate their members across city types and decide on the human capital investments per person for members by the type of city in which they live. For existing family members, current own human capital endowments are nontransferable, except to newborns. In this section of the paper, family decisions govern human capital accumulation, in the absence of formal markets for human capital, which we rule out for now under the usual ''no-slavery'' constraint.

For any dynasty, when a common form of utility from consumption per person, c, is used, the optimization problem is (without subscripting for t)

$$\max_{c,h_1,h_2,z} \int_0^\infty \left(\frac{c^{1-\sigma}-1}{1-\sigma}\right) e^{-(\rho-g)t} dt, \quad \sigma > 0, \rho > g, \tag{1}$$

$$\text{subject to } (a)\ \ P\dot{H} = ze^{gt}I_1 + (1-z)e^{gt}I_2 - Pce^{gt},$$

$$(b)\ \ H = ze^{gt}h_1 + (1-z)e^{gt}h_2,$$

$$(c)\ \ \dot{H} \geq 0; \quad \frac{\dot{h}_1}{h_1} + g \geq 0, \quad \frac{\dot{h}_2}{h_2} + g \geq 0.$$

In equation (1), given an initial normalized family size of 1, family size at time t is e^{gt}, and H is the family's human capital stock. A proportion z of family members are assigned to type 1 cities and $1 - z$ to type 2 cities. The terms I_1 and I_2 represent net incomes per worker earned by workers living in type 1 and type 2 cities, respectively, and h_1 and h_2 represent their human capital levels. Later we shall develop expressions for I_1 and I_2 to be either substituted into constraint a or added as additional constraints.

Constraint a, the equation of motion, states that the value of family human capital growth ($P\dot{H}$) is the difference between total family income ($ze^{gt}I_1 + [1-z]e^{gt}I_2$) and the value of consumption (Pce^{gt}). In constraint a it is assumed noncritically that human capital is formed by conversion of the consumption good produced in type 2 cities and sold at price P. Constraint b states that total family human capital is the sum of individual human capitals (h_1 and h_2) of members in type 1 (ze^{gt}) and type 2 ($[1-z]e^{gt}$) cities.

In our formulation of the dynasty's problem in equation (1), we make two important assumptions that limit transferability of human capital. Specifically, the first constraint in c ($\dot{H} \geq 0$) states that families can neither lend nor consume their human capital; conversion of the consumption good to human capital is irreversible. The second constraint in c states that, once installed, human capital is transferable as an endowment only to newborns in the same city type. The constraint $(\dot{h}_i/h_i) + g \geq 0$ states that the maximal percentage drop in human capital per member in a city type is the growth rate of their offspring. Neither of these constraints is binding in equilibrium, and results are also consistent with an additional constraint that human capital is specific to either a city or an industry, partially or fully nontransferable through migration across cities (see later).

Second, given that family members will generally earn different incomes I_1 and I_2 by city type, there must generally be intrafamily transfers across cities to maintain equality of consumption per member, c. In many countries, transfers from residents in large cities to relatives in smaller towns can amount to 10 percent of family income, a commonly given number supported, for example, by data in the Indian Statistical Institute's *Calcutta 1976: A Socio-economic Survey of Households with a Municipal Address*, where still remittances exceed 8 percent of personal income in a city several decades past its in-migration peak. In modern economies, people may not think so explicitly in these terms, despite evidence on large intrafamily income transfers (Gale and Scholz 1994). The rigidity of this formulation can be relaxed in several ways. First, rather than conceiving of patriarch-specified intrafamily income transfers, we can think of individuals in low–human capital cities investing formally in the human capital of family members in high–human capital cities and receiving an appropriate reimbursement. Second, dynasties can splinter as long as each splinter starts with the same stock of human capital per person (H/e^{gt}). Third, which follows from the first, if there is a more formal market for human capital, each family or family splinter can reside entirely in one type of city or the other, borrowing/investing in the human capital of families in the other type of city.

In order to proceed with the optimization problem in (1), it is necessary to determine the expressions for net real incomes, I_1 and I_2, which family members can earn in city types 1 and 2. To solve for them as well as to detail the nature of local knowledge or human capital spillovers, we turn to an analysis of production in cities, determination of city sizes (which affect the returns to human capital investment), and the like. Given that analysis, we can then return to the problem in (1) to study investment and migration decisions of families. They will determine the evolution of overall human capital levels, the formation of cities and their growth, and the size distribution of cities.

The Structure of Cities

Contemporaneous city formation and size determination involve a trade-off within cities between the benefits and costs of changing city sizes. We assume that production in a city occurs under ''localization'' economies of scale—own industry local external economies of scale. Contemporaneous efficiency of each firm is enhanced by having more firms in the same industry in a city, with which the firm communicates about what inputs to buy from whom, what product lines to emphasize, or how to organize production. This variant of a communications model may involve exogenous spillovers (Fujita and Ogawa 1982) or endogenous information exchange (Kim 1988). Over time, firm efficiency and the benefits of larger cities will be enhanced by local human capital accumulation and spillovers. When contemporaneous equilibrium city size is achieved, scale benefits are traded off against the higher internal commuting (and potentially congestion and pollution) costs per person of supporting large cities. We start by examining the structure of type 1 cities.

Production in Type 1 Cities

Consider a representative city of type 1. Each firm in the city is composed of one worker. Each period that worker decides how much to produce and how much to invest in private human capital accumulation. Having single worker-firms is a convenience, so that human capital spillovers exist only across firms, not within firms.[1] Output of firm i of the numeraire intermediate input X_1 (to be sold to type 2 cities) is given by

$$X_{1i} = D_1(n_1^{\delta_1} h_1^{\psi_1}) h_{1i}^{\theta_1}, \tag{2a}$$

where

$$W_{1i} = X_{1i}. \tag{2b}$$

In (2a), n_1 is employment in industry 1 in this city; h_1 is the average level of human capital of workers in the city; h_{1i} is the human capital

[1] If firms are multiworker, of course, in theory the firm could solve the internal coordination problem of each employee's investment decisions by imposing employment requirements. We did not add that complication here because it is not relevant to the problem.

of the worker in firm i; and δ_1 represents scale economies arising from the total volume of local communications that are proportional to n_1 (δ_1 is the elasticity of firm output with respect to total local employment, with own firm inputs held fixed). The elasticity of firm i's output with respect to the average level of human capital in the city is ψ_1, which represents the spillover benefits of local levels of human capital, our reduced-form specification (Romer 1986) of knowledge accumulation.[2] We use the average local level of human capital rather than total local human capital since scale economies are already captured in the $n_1^{\delta_1}$ term. The term $h_1^{\psi_1}$ could be thought of as representing the ''richness'' of the information spillovers $n_1^{\delta_1}$, based on the stock of ''local trade secrets.'' Equation (2b) tells us that a worker i's private income in a type 1 city, W_{1i}, is simply the output of that worker.

Given that all workers are inherently identical, in a symmetrical equilibrium as developed later, within city type 1, $h_{1i} = h_1$. Total city output then is $n_1 \times X_{1i}$ or

$$X_1 = D_1 n_1^{1+\delta_1} h_1^{\theta_1+\psi_1}. \tag{3}$$

The specification of technology assumes that human capital spillovers and scale externalities are purely localized. In addition, scale externalities are own industry, meaning that the presence of a different industry in the locality would not benefit the X_1 industry. In the analysis to follow, since agglomerating people into cities is costly on the commuting side, developers will form specialized cities, as in Henderson (1974). For the same population and commuting costs, a specialized city with greater scale per industry will have greater output per worker than a diversified city in which each separate industry operates at a lower scale.[3]

Commuting.—All production in a city occurs at a point, the central business district (CBD). Surrounding the CBD is a circle of residences, where each resident lives on a lot of unit size and commutes to the CBD (and back) at a constant cost per unit distance of τ (paid in units of type 1 city output). Adding in considerations of infrastruc-

[2] One could also adapt the elaborate structure in Romer (1990) in specifying the technology in city type 1. One method of doing this would be to retain X_1 as an intercity traded, competitive good but have it produced with nontraded machinery inputs, as well as labor and private human capital investment. Nontraded machinery inputs enter in X_1 production in Dixit-Stiglitz (1977) fashion, generating local economies of scale (Abdel-Rahman and Fujita 1990), and are sold locally under monopolistic competition. However, the local span or degree of diversity of these nontraded machinery inputs would increase with local human capital accumulation.

[3] With interindustry spillovers of communications, that argument is weakened, although not actually eliminated, provided that δ's (applied to, say, all industries' total local employment) vary by industry.

ture investments, congestion, pollution, and the like is critical for analyzing some features of urban growth but not the ones in this paper. For tractability, without loss of generality, we use the simplest standard version of the internal spatial structure of cities. In this version, equilibrium in the land market is characterized by a rent gradient, declining linearly from the CBD to the city edge, where rents (in the best alternative use) are zero. As city population expands, city spatial size, average commuting distances, and the rent gradient rise. Standard analysis gives us expressions for total city commuting costs and rents in terms of city population, where[4]

$$\text{total commuting costs} = bn_1^{3/2}, \tag{4}$$

$$\text{total land rents} = \tfrac{1}{2} bn_1^{3/2}, \tag{5}$$

and

$$b \equiv \tfrac{2}{3}\pi^{-1/2}\tau.$$

Equation (4) is a critical resource cost to the city, where average commuting costs ($bn_1^{1/2}$) rise with city size, with an elasticity of ½. That is the force limiting city sizes. Equation (5) constitutes the gross rental income of the city developer.

City developers.—Type 1 cities form in the competitive context of a large economy with many type 1 cities in the national land market. Each city is operated by its developer, who collects urban land rents, offers inducements to firms to locate in the city, and specifies city population (although people are free to move in equilibrium). Nationally, there are an unexhausted number of potential identical sites on which cities can form, and each developer controls only one site. This is a traditional formulation (e.g., Hamilton 1975; Scotchmer 1986), but the resulting solutions can be obtained in other ways. In a static context, Henderson and Becker (1998) show that this solution (1) is the only coalition-proof equilibrium and (2) will occur also in a model with only ‘‘self-organization,’’ where each existing city is governed by an autonomous local government. In a growth context, they show that an equivalent formulation is that de-

[4] An equilibrium in residential markets requires all residents (living on equal-sized lots) to spend the same amount on rent, $R(u)$, plus commuting costs, τu, for any distance u from the CBD. Any consumer then has the same amount left over to invest or spend on all other goods. At the city edge at a radius of u, rent plus commuting costs are τu_1 since $R(u_1) = 0$; elsewhere they are $R(u) + \tau u$. Equating those at the city edge with those amounts elsewhere yields the rent gradient $R(u) = \tau(u_1 - u)$. From this, we calculate total rents in the city to be $\int_0^{u_1} 2\pi u R(u)\,du$ (given lot sizes of one so that each ‘‘ring’’ $2\pi u du$ contains that many residents), or $\tfrac{1}{3}\pi\tau u_1^3$. Total commuting costs are $\int_0^{u_1} 2\pi u(\tau u)\,du = \tfrac{2}{3}\pi\tau u_1^3$. Given a city population of n and lot sizes of one, $n_1 = \tau u_1^2$ or $u_1 = \pi^{-1/2}n^{1/2}$. Substitution gives us eqq. (4) and (5).

velopers start or set up new cities to maximize profits, while existing cities are turned over to residents and perhaps governed by local autonomous governments.

Within a representative city, we specify the developer's optimization problem as a succession of contemporaneous profit maximization problems (cf. Deo and Duranton 1995). Formally, this assumption derives from the specification that there is only private human capital in the model, and developers cannot invest in human capital accumulation of residents (again, the no-slavery constraint). Later in the paper we shall devote attention to this issue, considering institutional specifications in which developers might invest in human capital to internalize local knowledge spillovers. For now we proceed with developers seeking to maximize contemporaneous profits. A developer's instantaneous profits are residential land rents (eq. [5]) less any transfer payments, T_1, to each worker-firm. The developer faces a free-migration constraint that each worker's net income (after rents and commuting costs are paid) equals the prevailing net real income available in national labor markets to workers in other type 1 cities, I_1. The developer announces city type and chooses city population, n_1, and transfer payments, T_1, to maximize current profits, or

$$\max_{n_1, T_1} \Pi_1 = \tfrac{1}{2} b n_1^{3/2} - T_1 n_1 \tag{6}$$

$$\text{subject to } W_1 + T_1 - \tfrac{3}{2} b n_1^{1/2} = I_1,$$

where from (1a) $W_1 = D_1 n_1^{\delta_1} h_1^{\theta_1 + \psi_1}$, given symmetry within the city. In the constraint the first term is private income per worker-firm and the third term is rent plus commuting costs per resident anywhere in the city from (4) and (5), so the total left-hand side is net real income earned in the city. Solving (6), substituting for T_1 back into Π_1 (eq. [6]), and setting $\Pi_1 = 0$ (through folk theorem free entry of developers/cities in national land markets) together yield basic urban results:

$$T_1 = \tfrac{1}{2} b n_1^{1/2} \tag{7}$$

and

$$n_1 = (\delta_1 2 b^{-1} D_1)^{2/(1-2\delta_1)} h_1^{2\epsilon_1} \tag{8}$$

for

$$\epsilon_1 \equiv \phi_1 + \frac{\psi_1}{1 - 2\delta_1}; \quad \phi_1 \equiv \frac{\theta_1}{1 - 2\delta_1} < 1.$$

The first result in equation (7) reflects the Henry George theorem (Flatters et al 1974; Stiglitz 1977). Total transfers to firms ($T_1 n_1$) equal total urban land rents ($\frac{1}{2} b n_1^{3/2}$), where the transfer per firm closes exactly the gap between private marginal product (eq. [2]) and social marginal product due to enhanced scale benefits when a worker-firm enters a city (the gap is $\delta_1 W_1$). Developers (or local governments) have the incentive to subsidize entry to their cities, internalizing the benefits of local scale externalities. The second result in equation (8) tells us that equilibrium city size is a function of scale and other parameters. It is also a function of human capital per worker, a new result of focus in the paper. Second-order conditions and equation (8) reveal the parameter restriction $\delta_1 < \frac{1}{2}$, necessary to have multiple type 1 cities in the economy. Equation (8) shows that city sizes increase as the scale elasticity, δ_1, rises toward the commuting cost of elasticity, $\frac{1}{2}$, from equation (4). For $\delta_1 > \frac{1}{2}$, all X_1 production would occur in just one city because marginal scale benefits of increasing city size would always outweigh marginal costs. Finally, given that equation (8) is satisfied under the constraint to (6), city sizes will be "self-enforcing." Families cannot gain by moving another person into a type 1 city (away from another type 1 or type 2 city [see below]). So equilibria will be free-mobility ones. This result will also be apparent in the solution to equation (14) below.

In terms of the relationship between city size and human capital, $2\epsilon_1$ defines the elasticity of city size with respect to human capital per worker, which is increasing in the private (θ_1) and external (ψ_1) elasticities of productivity with respect to human capital. The term ϵ_1 is decomposed into a private return portion ϕ_1 and an externality return portion $\psi_1/(1 - 2\delta_1)$, according to the private, θ_1, and spillover, ψ_1, returns to human capital. Regularity of equilibrium solutions requires $\phi_1 < 1$. The value of ϵ_1 rises as the degree of scale economies, δ_1, rises toward the commuting resource cost elasticity, $\frac{1}{2}$. Scale benefits augment human capital returns. As we shall see next in equation (9), ϵ_1 is the elasticity of net income in a city with respect to average human capital levels. With human capital accumulation, not only do incomes rise, but city sizes as well.

For later use, we solve for income, W_1, and net real income, I_1, by substitution of (7) and (8) into (5) and (2), as well as city output X_1. For Q_1 a parameter cluster,[5]

$$I_1 = (1 - 2\delta_1) W_1 = Q_1 h_1^{\epsilon_1}. \tag{9}$$

[5] The parameter cluster $Q_1 \equiv (\delta_1 2 b^{-1} D_1)^{1/(1-2\delta_1)} b (2\delta_1)^{-1} (1 - 2\delta_1)$ and $X_1 = \{[Q_1/(1 - 2\delta_1)]^{(1+\delta_1)/\delta_1} D_1^{-\delta_1}\} h_1^{3\epsilon_1}$.

Type 2 Cities

Type 2 cities specialize in production of the economy's consumption good, sold in competitive national markets at a price P. A single worker-firm's output is

$$X_{2j} = D_2(n_2^{\delta_2} h_2^{\psi_2}) h_{2j}^{\theta_2} x_{1j}^{1-\alpha}, \tag{10}$$

where external scale ($n_2^{\delta_2}$) and human capital ($h_2^{\psi_2}$) terms correspond to equation (2) for X_{1i}; h_{2j} is worker j's human capital; and x_{1j} is the jth firm's use of imported intermediate inputs of type 1 cities. Profits for a firm are $PX_{2j} - x_{1j}$. Maximizing and substituting in $PX_{2j} - x_{1j}$ for the choice of x_{1j} gives the residual return to the worker-firm:

$$W_2 = \alpha(1-\alpha)^{(1-\alpha)/\alpha} D_2^{1/\alpha} P^{1/\alpha} (n_2^{\delta_2} h_2^{\psi_2})^{1/\alpha} h_{2j}^{\theta_2/\alpha}. \tag{11}$$

As for type 1 cities, developers of type 2 cities announce city type and choose T_2 and n_2 to max $\Pi_2 = \frac{1}{2} b n_2^{3/2} - T_2 n_2$ subject to $W_2 + T_2 - \frac{3}{2} b n_2^{1/2} = I_2$. The term W_2 is given in (11) for $h_{2j} = h_2$ under internal city symmetry, and commuting and rents for a representative type 2 city are derived as for type 1 cities. Note that commuting costs and rents are paid and enumerated in units of X_1, the numeraire good. We solve this problem as before, maximizing, setting Π_2 equal to zero, and solving for the Henry George result, $T_2 = \frac{1}{2} b n_2^{1/2}$. With substitutions, we have equations corresponding to (8) and (9) defining equilibrium city size and net real income:[6]

$$n_2 = C_2 P^{1/[(\alpha/2)-\delta_2]} h_2^{2\epsilon_2}, \quad \delta_2 < \frac{\alpha}{2}, \tag{12}$$

where

$$\epsilon_2 \equiv \phi_2 + \frac{\psi_2}{\alpha - 2\delta_2}, \quad \phi_2 \equiv \frac{\theta_2}{\alpha - 2\delta_2} < 1,$$

and

$$I_2 = (\alpha - 2\delta_2)\alpha^{-1} W_2 = Q_2 P^{1/(\alpha - 2\delta_2)} h_2^{\epsilon_2}. \tag{13}$$

[6] Firm level

$$x_1 = (1-\alpha) Q_2 (\alpha - 2\delta_2)^{-1} P^{1/(\alpha-2\delta_2)} h_2^{\epsilon_2},$$
$$C_2 \equiv [(1-\alpha)^{(1-\alpha)/\alpha} \delta_2 2 b^{-1} D_2^{1/\alpha}]^{\alpha/[(\alpha/2)-\delta_2]},$$

and

$$Q_2 \equiv [(1-\alpha)^{(1-\alpha)/\alpha} \delta_2 2 b^{-1} D_2^{1/\alpha}]^{\alpha/(\alpha-2\delta_2)} b(2\delta_2)^{-1}(\alpha - 2\delta_2).$$

While these expressions have properties similar to those for type 1 cities, they contain the relative price P. We need first to determine migration and human capital investment decisions based on the family's problem in (1), which we are now ready to solve, given that we know I_1 and I_2. Then we can solve for P in national output markets and proceed to growth properties.

Investment and Migration Decisions

Given the family's dynamic optimization in (1), we form the Hamiltonian, ignoring for now constraints c, which, as we indicate later, are never binding. A representative family's Hamiltonian is

$$\begin{aligned}\max_{c,z,h_1,h_2,H} \mathcal{L} = {} & \frac{c^{1-\sigma}-1}{1-\sigma} e^{-(\rho-g)t} \\ & + \lambda_1[ze^{gt}I_1P^{-1} + (1-z)e^{gt}I_2P^{-1} - ce^{gt}] \\ & + \lambda_2[H - ze^{gt}h_1 - (1-z)e^{gt}h_2].\end{aligned} \tag{14}$$

As written, the problem in (14) is incomplete because we need to substitute into the first constraint for I_1 and I_2 for an individual family. As perceived by family i, for workers in city type 1, on the basis of equations (6) and (2a), $I_{1i} = W_{1i} + T_1 - \frac{3}{2} bn_1^{1/2}$ for $W_{1i} = D_1(n_1^{\delta_1}h_1^{\psi_1})h_{1i}^{\theta_1}$ and T_1, h_1, and n_1 perceived as fixed by the family. Thus $\partial I_{1i}/\partial h_{1i} = \theta_1 W_{1i}/h_{1i}$. Then if we impose symmetry ($h_{1i} = h_1$) and use equations (8) and (9), in equilibrium, the value of $\partial I_{1i}/\partial h_{1i} = \theta_1/(1-2\delta_1)I_1h_1^{-1} = \phi_1 I_1 h_1^{-1}$. Similarly in city type 2, $\partial I_{2i}/\partial h_{2i} = \theta_2/(\alpha - 2\delta_2)I_2h_2^{-1} = \phi_2 I_2 h_2^{-1}$. The first-order conditions for (14) imposing symmetry and doing the substitutions above following differentiation are

$$\frac{\partial \mathcal{L}}{\partial c} = c^{-\sigma}e^{-(\rho-g)t} - \lambda_1 e^{gt} = 0, \tag{15a}$$

$$\frac{\partial \mathcal{L}}{\partial z} = e^{gt}[\lambda_1(I_1P^{-1} - I_2P^{-1}) + \lambda_2(-h_1 + h_2)] = 0, \tag{15b}$$

$$\frac{\partial \mathcal{L}}{\partial h_1} = ze^{gt}(\lambda_1\phi_1 I_1 h_1^{-1}P^{-1} - \lambda_2) = 0, \tag{15c}$$

$$\frac{\partial \mathcal{L}}{\partial h_2} = (1-z)e^{gt}(\lambda_1\phi_2 I_2 h_2^{-1}P^{-1} - \lambda_2) = 0, \tag{15d}$$

$$\frac{\partial \mathcal{L}}{\partial H} = -\dot{\lambda}_1 = \lambda_2. \tag{15e}$$

The transversality condition requires

$$\lim_{t\to\infty} [\lambda_1(t)H(t)] = 0. \tag{15f}$$

In (15c) and (15d), families allocate human capital across city types to equalize private returns on investment. When the two are combined, $I_1/I_2 = (\phi_2/\phi_1)h_1/h_2$, which then, when combined with the result from solving λ_2/λ_1 in (15b) and (15c), yields

$$h_1 = \left[\frac{\phi_1(1-\phi_2)}{\phi_2(1-\phi_1)}\right]h_2, \tag{16a}$$

$$I_1 = \left(\frac{1-\phi_2}{1-\phi_1}\right)I_2. \tag{16b}$$

Note the time-invariant ratios of h_1/h_2 and I_1/I_2. To proceed further to solve for z, the relative allocation of family members by city type, we need to examine equilibrium in national markets.

National Market Equilibrium

Equilibrium in national output markets requires a balance of trade among cities, so national demand for (by type 2 cities) and supply of (by type 1 cities) X_1 are equalized:[7]

$$z = \frac{(1-\phi_1)(1-\alpha+2\delta_2)}{(1-\phi_1)(1-\alpha+2\delta_2)+(1-\phi_2)(\alpha-2\delta_2)}. \tag{17}$$

Any family's proportion of workers, z, going to a type 1 city in equilibrium is invariant to h (h_1 or h_2) and is constant over time. All workers

[7] National supply of X_1 is m_1X_1, where m_1 is the number of type 1 cities. The m_2 type 2 cities import X_1 as an intermediate input x_1, and X_1 is used to produce commuting (eq. [4]) in both types of cities. Trade balance requires $m_1X_1 = m_2n_2x_1 + m_1(bn_1^{3/2}) + m_2(bn_2^{3/2})$. Then imposing symmetry across dynasties nationally, so that each dynasty sends the same proportion of workers to each type of city, we know at any instant that $z = m_1n_1/N$ and $1 - z = m_2n_2/N$, where N is national population. When we rearrange so that demand equals supply, $m_1(X_1 - bn_1^{3/2}) = m_2n_2[(x_1/n_2) + bn_2^{1/2}]$. From (6), (1), and (7), $n_1I_1 = X_1 - bn_1^{3/2}$. From (11), (12), and n. 6, we know that $(x_1/n_2) + bn_2^{1/2} = I_2(1-\alpha+2\delta_2)/(\alpha-2\delta_2)$. Combining these relationships gives $m_1n_1I_1 = m_2n_2I_2(1-\alpha+2\delta_2)/(\alpha-2\delta_2)$. Substituting this and eq. (16) with the expressions for z and $1 - z$ gives (17).

once assigned a city type never need to change that type. Migration typically involves only assignment of newborns, especially to new cities (see below). We can also solve for the number of type 1 and type 2 cities, m_1 and m_2, as a function of national population, N, at that instant:[8]

$$m_1 = zNn_1^{-1}, \quad m_2 = (1 - z)Nn_2^{-1}. \tag{18}$$

Having solved for z, which reflects migration decisions, we can solve for capital usages, h_1 or h_2, as functions of family capital stock per person, h. Given $h = He^{-gt}$ from (1b), $h = zh_1 + (1 - z)h_2$. Substituting in (16) and (18) yields

$$h_2 = \frac{\phi_2}{1 - \phi_2} Kh, \quad h_1 = \frac{\phi_1}{1 - \phi_1} Kh, \tag{19}$$

where

$$K \equiv \frac{(1 - \phi_1)(1 - \alpha + 2\delta_2) + (1 - \phi_2)(\alpha - 2\delta_2)}{\phi_1(1 - \alpha + 2\delta_2) + \phi_2(\alpha - 2\delta_2)}.$$

Equation (17) directly gives us an unchanging relative allocation of family members by city type. In the Appendix we note that the constraint $\dot{H} \geq 0$ is satisfied along stable growth paths. From (16) and (22) below, when we time-differentiate, $(\dot{h}_1/h_1) + g = (\dot{h}_2/h_2) + g = (\dot{h}/h) + g = \dot{H}/H \geq 0$. Human capital grows in parallel at the same rate in the two types of cities. The only capital transfers need go from each worker type to its own children. In equilibrium, human capital can be nontransferable across existing people and specific to a technology (either X_1 or X_2). In (1c) constraints are never binding.

Finally, by combining various relationships, we get[9]

$$P = Qh^{(\epsilon_1 - \epsilon_2)(\alpha - 2\delta_2)}, \tag{20}$$

where

$$Q \equiv \left[\frac{\phi_1 Q_1}{\phi_2 Q_2} \frac{\left(\dfrac{\phi_1}{1 - \phi_1}\right)^{\epsilon_1 - 1}}{\left(\dfrac{\phi_2}{1 - \phi_2}\right)^{\epsilon_2 - 1}} K^{\epsilon_1 - \epsilon_2} \right]^{\alpha - 2\delta_2}.$$

[8] Combine (17) with $z = m_1 n_1/N$.
[9] We combine (16) with $z = m_1 n_1/N$ and $1 - z = m_2 n_2/N$ and do substitutions.

In (20), as h grows, the relative price of the consumption good rises if $\epsilon_1 > \epsilon_2$. Not surprisingly, P rises so that the consumption good becomes more expensive if the elasticity of income in the numeraire good city with respect to human capital exceeds that in the consumption-type city.

The results in this section are summarized in the following proposition.

PROPOSITION 1. Over time, the equilibrium allocation of resources across cities involves the following characteristics: (*a*) The ratios of human capital and income per person, h_1/h_2 and I_1/I_2, are time-invariant. This implies persistent measured real income inequality, where $I_1 > (<)\ I_2$ and $h_1 > (<)\ h_2$ iff $\phi_1 > (<)\ \phi_2$, where ϕ_i is the private return on income to human capital investment in city type i. (*b*) The relative allocation of population across cities is time-invariant. (*c*) The price P of X_2, the consumption good, rises (falls) with human capital accumulation iff $\epsilon_1 > (<)\ \epsilon_2$, where ϵ_i is the social return to human capital investment in city type i.

Note on the issue of inequality that not only do real incomes, I_1 and I_2, differ across city types, given different human capital levels, but nominal incomes, W_1 and W_2, differ given, additionally, cost-of-living differences across cities, so $W_1/W_2 = (\alpha - 2\delta_2)(1 - 2\delta_1)^{-1}\alpha^{-1}(I_1/I_2)$. Henderson (1988) presents and reviews evidence suggesting that cost-of-living differences across cities typically exceed 100 percent within a country. With real income differences, in our formulation given intrafamily transfers, real incomes net of the opportunity cost of capital are equal across city types (given the choice of z) and consumption per worker is also the same across cities. However, in Section IV, we shall note extensions in which consumption and income net of human capital costs can also diverge across city types.

We can now solve for urban and economic growth features of the economy.

II. Growth Properties

Growth properties are divided into aspects of urban growth, including empirical evidence, and economic growth properties.

Urban Growth

Although human capital levels per person employed in each type of city differ at any instant, as we just saw, human capital in each type of city grows at the same rate, or $\dot{h}_1/h_1 = \dot{h}_2/h_2 = \dot{h}/h$. Then in the city size equations (8) and (12) combined with (20) for P, we know that

$$\frac{\dot{n}_2}{n_2} = \frac{\dot{n}_1}{n_1} = 2\epsilon_1 \frac{\dot{h}}{h}. \tag{21}$$

Individual city sizes grow with human capital accumulation at a rate $2\epsilon_1$ times the rate of human capital accumulation. Recall that ϵ_1 is the elasticity of income with respect to human capital levels in city type 1. Below, if the economy experiences steady-state growth, ϵ_1 may be close to one and city sizes grow at approximately twice the rate of human capital accumulation. Through externalities raising the marginal benefits of adding population to cities relative to the marginal costs, human capital accumulation enhances productivity per worker directly and indirectly sufficiently to cause cities to grow at about twice the rate of capital accumulation.

What about growth in the number of cities, m_1 and m_2? From (18), which gives m_1 and m_2, for example, $\dot{m}_1/m_1 = (\dot{N}/N) - (\dot{n}_1/n_1)$, where national population growth $(\dot{N}/N)$ is g and $\dot{n}_1/n_1$ is given by (21). Thus

$$\frac{\dot{m}_1}{m_1} = \frac{\dot{m}_2}{m_2} = g - 2\epsilon_1 \frac{\dot{h}}{h}. \tag{22}$$

City numbers increase with human capital accumulation if the rate of individual city size growth fueled by human capital accumulation is not high enough to accommodate the expanding national population growth rate. Regardless, equations (21) and (22) imply by inspection the following proposition.

Proposition 2. Individual city sizes grow at a rate proportional to the rate of human capital accumulation. Given that relative sizes and numbers of types of cities are time-invariant, urban growth across city types is parallel, maintaining a constant relative size distribution of cities.

Empirical Evidence

Using panel data for the United States, we test in this subsection whether individual city growth rates are closely tied to growth rates in local educational attainment and inferred human capital spillovers. We estimate the relationship between growth in city sizes and growth in local human capital levels, using decade data on U.S. metro areas for 1940–90. The sample includes the 318 metropolitan statistical areas (MSAs) in the 48 contiguous states as of 1990 (we

TABLE 1

Relationship between City Size and Local Human Capital Levels

	ln (MSA Population) (1)	(2)	(3)	ln (MSA Urban Population)[a] (4)
Constant	11.81*	11.50*	11.70*	11.59*
	(.027)	(.043)	(.102)	(.035)
Percentage college educated	2.78*	2.54*		4.05*
	(.343)	(.334)		(.456)
Percentage high school educated	. . .	. . .	.476*	. . .
			(.215)	
Ratio: manufacturing employment/population over 25[b]	. . .	2.07*	2.16*	. . .
		(.227)	(.204)	
Time and MSA fixed effects	yes	yes	yes	yes
Observations	318	318	318	
Total observations	1,590	1,590	1,908	1,319

Note.—The means and standard deviations of percentage college and percentage high school and ratio in manufacturing are, respectively, (.117, .072), (.503, .194), and (.155, .085). Standard errors are in parentheses.

[a] Sample is MSAs with an urban population over 50,000.

[b] For 1940–70, employment is total employment in manufacturing, whereas for 1980 and 1990 it is the civilian labor force in manufacturing.

* Significant at the 5 percent level.

look at these same MSAs back through time to 1940).[10] We hold the geographic metro area definitions fixed, on the basis of 1990 definitions. We test formulations for the whole sample (all 318 MSAs in all available decades) and for a sample requiring each MSA in each decade to have an urban population over 50,000 (from contemporaneous definitions of urban). The estimation method imposes MSA and year fixed effects, so in essence we are examining the effect on individual city sizes of time variation in their human capital, controlling for innate city characteristics and national time trends.

We do not have an MSA measure of average human capital per se, just related information on educational attainment of the population: percentage adults (over 25 years) with four or more years of college and percentage with high school graduation. The college measure is what we view as the key measure for the second half of the century; it is unavailable in our data for 1960, whereas the high school measure is available for all six decades. We report results for both measures.

Results are in table 1. We focus on the simple fixed-effects regres-

[10] The 318 MSAs are defined from 743 urban counties on the basis of ‘‘common denominator’’ county definitions for 1940–90. That is, during that time period, some counties are either split apart or joined together. For consistency over time, counties are given time-invariant definitions, based on the definition, respectively, before they split apart or after they are joined together.

sion in column 1, based on the panel relationship between MSA population and percentage adults with college. Column 2 is also relevant, allowing us to impose a crude control for changes in city function, or type, by controlling for industrial composition of the city labor force. Column 3 shows a result for percentage adults with high school, and column 4 shows a result measuring size by urban rather than total population and restricting the sample to MSAs with an urban population of over 50,000 in each decade.

In column 1, when we control for innate city characteristics and national city size trends, a one-standard-deviation increase in a city's percentage college educated increases city size by 20 percent. Or, for the constant term (1950), a city at plus one and one-half standard deviations of college educated is 82 percent larger than a city at minus one and one-half standard deviations of education, when we control for innate city characteristics. The results for high school are smaller but significant. Controlling for industrial composition has little effect in column 2 but does enhance the high school education result in column 3. In column 4, looking at size as measured by urban population, as opposed to total population, leads to an increase in the effect of percentage college. Overall, the evidence supports strongly the theoretical result that individual city size growth rates are related to individual local human capital growth rates.

Other Aspects of the Urban Growth Process

While this urban growth process seems simple, the underlying institutional and economic reality can be quite complex. Formation of an appropriate number of new cities at any instant conceptually requires "large" agents such as developers who set up new cities in a conducive institutional framework (cf. Krugman 1993). In the absence of such agents, cities in general will tend to be too large and too few in number. Henderson and Becker (1998) argue that part of the problem of top-heavy urban development in some developing countries may be central government hindrance of effective functioning of land markets and local governments. Fortunately, the existence and widespread operation of developers who set up new cities seem, at least for the United States, to be a fact (e.g., Garreau 1991). Formation of new cities in and of itself efficiently limits the contemporaneous sizes of existing cities; and, in theory, autonomous local governments in existing cities have the incentives to offer appropriate local subsidies, T_1 (eq. [7]) and the corresponding T_2, to local businesses (Henderson and Becker 1998). In summary, the process works if new cities are started by developers and existing cities have traditional United States–style local governments.

From equations (21) and (22) it appears that once type 1 cities are set up, they stay and grow as type 1 cities, and the same for type 2. At each instant, new type 1 and type 2 cities form with newborn people. In Black and Henderson (1997), this does not appear to be the way the process actually works. If, say, type 1 cities are smaller than type 2 cities, empirically, new cities coming into existence between 1900 and 1950 all appear to be smaller type 1 cities. Additional type 2 cities arise when type 1 cities transform into type 2 cities. Given that type 1 and type 2 cities operate with different human capital levels per person, converting type 1 cities must upgrade or downgrade human capital levels. With specific human capital, transforming the human capital base would require migration: exit of type 1 workers from transforming type 1 cities to new type 1 cities and entry of type 2 workers (who could be newborns).

The big question is, Why do type 1 cities transform to type 2 cities to accommodate growth in numbers of type 2 cities, rather than entirely new cities of both types forming? There seem to be two potential explanations, both beyond the formal scope of this paper. Nevertheless, it is instructive to consider them. We assumed that all potential city sites in the economy are identical: offer identical (unspecified) natural public amenities such as climate, coastal location, harbor facilities, and so forth. In reality, there is a spectrum of site qualities. Models that start to deal with this problem (Upton 1981; Henderson 1988, pp. 71–73) appear to have two features to equilibrium. The best sites are occupied first and the best sites go to bigger types of cities, which can bid more for the amenities. Here that means that as the number of cities grows, additional, bigger type 2 cities outcompete existing type 1 cities (which initially got reasonable quality sites) for the sites they are on, transforming the industrial base at those sites. New type 1 cities form on the lowest-quality sites occupied to date.

The second explanation is institutional, although it can be specified to have market foundations (Helsley and Strange 1993). Developers who start new cities have either or both limited financial resources and ability to assemble large pieces of land. It is thus ‘‘easier’’ for them to start new, smaller types of cities. Later with growth these initial smaller types may transform into bigger types of cities. In both cases of the site quality and the limited size developer models, an issue concerns the transformation process. To enact mass conversion of production in a city to another type involves large-scale movement/conversion of firms, which is not readily attained through atomistic behavior. With scale economies, local developers or local governments or both are needed to facilitate timely transformation (see Rauch 1993*a*).

Economic Growth

The final part to urban evolution is to solve for growth paths in the economy. Once in place, the analysis is fairly standard and our treatment is very brief. We need to study the representative family's evolution of consumption and human capital.[11] Solving the model yields equations for growth rates of consumption and human capital per person:

$$\gamma^c \equiv \frac{\dot{c}}{c} = \frac{1}{\sigma}(Ah^{\epsilon-1} - \rho) \tag{23}$$

and

$$\gamma^h \equiv \frac{\dot{h}}{h} = Bh^{\epsilon-1} - ch^{-1} - g, \tag{24}$$

where

$$\epsilon \equiv \epsilon_1[1 - (\alpha - 2\delta_2)] + \epsilon_2(\alpha - 2\delta_2) > 0,$$

$$A \equiv \frac{\phi_1 Q_1}{Q}\left(\frac{\phi_1}{1-\phi_1}\right)^{\epsilon_1 - 1} K^{\epsilon_1 - 1},$$

$$B \equiv A[\phi_1(1 - \alpha + 2\delta_2) + \phi_2(\alpha - 2\delta_2)]^{-1}, \quad A < B.$$[12]

Growth properties of the model depend on ϵ, which equals a weighted average of the elasticities of net real incomes with respect to human capital, ϵ_1 and ϵ_2, in the two city types. In the analysis to follow we distinguish between two cases: steady-state growth in which $\epsilon = 1$ and steady-state levels in which $\epsilon < 1$. For $\epsilon = 1$, either ϵ_1 or ϵ_2 is greater than one or both equal one. Global stability and uniqueness are discussed in the Appendix. The basic proposition follows.

PROPOSITION 3. If $\epsilon = 1$, the economy achieves steady-state growth, where consumption and human capital grow at the rate

[11] For consumption, we first time-differentiate (15a) and combine with (15c) and (15e) to get $\gamma^c \equiv \dot{c}/c = (1/\sigma)(\phi_1 I_1 h_1^{-1} P^{-1} - \rho)$. For the human capital growth path, we focus on the average level of human capital per member, h, where $\dot{h}/h = (\dot{H}/H) - g$; so from (14a), $\gamma^h \equiv \dot{h}/h = zI_1P^{-1}h^{-1} + (1-z)I_2P^{-1}h^{-1} - ch^{-1} - g$. Into these equations we substitute I_1 and I_2 from (9) and (13), h_1 and h_2 from (19), and P from (20).

[12] Given $z > 0$, $B/A = [\phi_1(1 - \alpha + 2\delta_2) + \phi_2(\alpha - 2\delta_2)]^{-1} > 1$; $z > 0$ requires $1 > \phi_1(1 - \alpha + 2\delta_2) + \phi_2(\alpha - 2\delta_2)$, because the denominator of z can be written as $1 - [\phi_1(1 - \alpha + 2\delta_2) + \phi_2(\alpha - 2\delta_2)]$.

$(A - \rho)\sigma^{-1}$ and city sizes grow at $2\epsilon_1$ times this rate. If $\epsilon < 1$, the economy converges to steady-state levels of consumption and human capital and cities achieve a stationary size.

If $\epsilon = 1$, by inspection of (23) the steady-state growth rate of c is $(A - \rho)/\sigma$. By differentiation of (24), for γ^h to be constant, the steady-state growth rate of h must equal that of c. Denote the steady-state growth rates as $\bar{\gamma}^c$ and $\bar{\gamma}^h$; then

$$\bar{\gamma}^c = \bar{\gamma}^h = \frac{A - \rho}{\sigma}. \tag{25}$$

Positive steady-state growth with bounded utility and satisfaction of transversality conditions require[13]

$$A - \rho > 0, \tag{26a}$$

$$\frac{1 - \sigma}{\sigma} A + g - \frac{\rho}{\sigma} < 0. \tag{26b}$$

With steady-state growth, from equation (21), city sizes grow at a rate $2\epsilon_1 \bar{\gamma}^h$ indefinitely. City numbers increase as long as the individual city population growth rate is less than the national population growth rate.

If $\epsilon < 1$, to solve for steady-state levels, we set $\gamma^h = \gamma^c = 0$ and solve

$$\begin{aligned} \bar{h} &= \left(\frac{A}{\rho}\right)^{1/(1-\epsilon)}, \\ \bar{c} &= \left(\frac{A}{\rho}\right)^{1/(1-\epsilon)} \left(\frac{B\rho}{A} - g\right). \end{aligned} \tag{27}$$

Positive consumption requires $(B\rho/A) - g > 0$, which is guaranteed given $B > A$ and $\rho > g$. In the Appendix we note that $\bar{h}$, $\bar{c}$ exhibits local saddle path stability, and convergence occurs along a globally stable arm. Along the stable arm, $\dot{H} > 0$. As one moves upward along the stable arm, h is increasing and, hence, so are city sizes. However, at steady-state levels, since h growth ceases, city sizes stagnate.

[13] For bounded utility,

$$\lim_{t\to\infty} \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho - g)t} = 0.$$

Given that, from (25), $c = c_0 \exp[(A - \rho)/\sigma]t$, the limit requires (26b), and similarly from transversality $\lim_{t\to\infty} \lambda_1(t) H(t) = 0$, where $H(t) = h(t)e^{gt}$ and $\lambda_1(t) = c^{-\sigma}e^{-\rho t}$ from (15a). Evaluating again requires (26b) to be satisfied.

III. Efficiency and Urban Institutions

The equilibrium growth outcome is not optimal. The problem lies with the externalities involved in human capital accumulation decisions and with the corresponding population allocation decisions made by families. The key is the gap between ϵ_i and ϕ_i, where $\epsilon_1 = \phi_1 + [\psi_1/(1 - 2\delta_1)]$ and $\epsilon_2 = \phi_2 + [\psi_2/(\alpha - 2\delta_2)]$. Families invest on the basis of net private returns, ϕ_i, rather than the net social returns ϵ_i, ignoring spillover returns ψ_i.

In the Appendix, we specify a national social planner's optimization problem based on equation (1) for a representative family, where the planner accounts for overall social marginal returns to human capital investment in cities. We examine a simple optimization problem in which $\epsilon_1, \epsilon_2 < 1$, so $\epsilon < 1$, a case involving steady-state levels. Other values of ϵ_i involve limit cases, where characterizing solutions and comparing them to equilibrium ones are beyond the scope of the paper. However, the case we examine will suffice to analyze urban institutions.

The solution to the planner's problem has two key aspects. First, the time-invariant ratios, h_1/h_2 and $z/(1 - z)$, are different from the equilibrium ratios in equations (16) and (17), and hence so are n_1/n_2, m_1/m_2, and I_1/I_2. Of particular interest are

$$\frac{\overset{*}{h}_1}{\overset{*}{h}_2} = \frac{\epsilon_1(1 - \epsilon_2)}{\epsilon_2(1 - \epsilon_1)},$$
$$\frac{\overset{*}{I}_1}{\overset{*}{I}_2} = \frac{1 - \epsilon_2}{1 - \epsilon_1}, \tag{28}$$
$$\overset{*}{z} = \frac{(1 - \epsilon_1)(1 - \alpha + 2\delta_2)}{(1 - \epsilon_1)(1 - \alpha + 2\delta_2) + (1 - \epsilon_2)(\alpha - 2\delta_2)}.$$

Combining results in the Appendix with (28), we get the following proposition.

Proposition 4. With an efficient allocation of resources, compared to the equilibrium, steady-state levels of consumption and capital per person are higher. In terms of allocations to type 1 versus type 2 cities, h_1/h_2 and I_1/I_2 rise (fall) respectively iff

$$\frac{\epsilon_1}{\epsilon_2}\frac{1 - \epsilon_2}{1 - \epsilon_1} > (<) \frac{\phi_1}{\phi_2}\frac{1 - \phi_2}{1 - \phi_1}$$

and

$$\frac{1 - \epsilon_2}{1 - \epsilon_1} > (<) \frac{1 - \phi_2}{1 - \phi_1}.$$

The proof of proposition 4 is in the Appendix. But the intuition is straightforward. In proposition 4, since the social returns to capital exceed the private, the optimal solution involves greater capital accumulation. In that solution, the ratios of incomes and capital in the two types of cities depend on social, not private, returns. But whether, say, I_1/I_2 rises does not simply depend on just whether $\psi_1/(1-2\delta_1) > \psi_2/(\alpha-2\delta_2)$, but depends on the initial position of ϕ_1 versus ϕ_2. So I_1/I_2 rises if $(1-\phi_2)\psi_1/(1-2\delta_1) > (1-\phi_1)\psi_2/(\alpha-2\delta_2)$. In comparisons of relative city sizes, what happens to n_1/n_2 (after substitution for h_1/h_2) involves a complex expression.

Beyond proposition 4 and equation (28), we do not focus in the text on detailed comparisons of equilibrium versus efficient outcomes, which follow a predictable pattern (ϕ_i's are replaced by ϵ_i's in key expressions). Rather the focus is to ask whether institutions in urban settings could generate efficient outcomes. In theory, the answer is yes; but implementation is so problematical that the efficient solution was not presented as our base case. Let us first see how the process could work and then discuss its problems.

To generate efficient outcomes, developers or autonomous local governments must be able to specify the levels of h_i required for each entrant to their city and must be motivated to set that h_i at the efficient level for every entrant. The problems in specification and incentives to be discussed below do not derive from whether there is market pricing of human capital per se. To make that point and to shorten the exposition, we assume for now that there is an effective national market in human capital that might be generated through student loan programs, where human capital can be borrowed at a prevailing rental rate in national markets. We explore the issues for cities operating in such a market in two specifications. The basic problem a developer faces in such a market in the second specification is the same as when there is no human capital market, but a developer specifies a local human capital requirement, accounting for the shadow cost to residents of increased capital.[14]

With an effective market price for human capital, consider two possible ways of rewriting the developer's optimization problem for a representative type 1 city:

[14] Given a frictionless economy, in general a barter portion of the economy can operate with the same outcomes as though there were a competitive market in that portion (e.g., application of Debreu and Scarf [1963]). That is, here developers can treat the expressions in eq. (A4) in the Appendix as opportunity costs, equivalent to r. The indivisibilities present (e.g., individuals cannot be split across cities) are not a problem, given that families allocate fractions of their whole across cities with divisible human capital allocations.

$$\max_{T_1, n_1, h_1} \tfrac{1}{2} b n_1^{3/2} - T_1 n_1 - r h_1 n_1 + \lambda (D_1 n_1^{\delta_1} h_1^{\psi_1+\theta_1} + T_1 - \tfrac{3}{2} b n_1^{3/2} - I_1), \tag{29a}$$

$$\max_{T_1, n_1, h_1} \tfrac{1}{2} b n_1^{3/2} - T_1 n_1 + \lambda (D_1 n_1^{\delta_1} h_1^{\psi_1+\theta_1} + T_1 - \tfrac{3}{2} b n_1^{3/2} - r_1 h_1 - I_1). \tag{29b}$$

In the first, developers borrow h_1 in national markets for their workers (either h_1 per worker as specified or, at the margin, to supplement private choices), at prevailing rental rates, from families supplying capital to national markets. In the second, developers recognize that individuals must borrow in national markets to achieve a specified h_1, keeping income generated ($D_1 n_1^{\delta_1} h_1^{\psi_1+\theta_1}$) but repaying the loan (at a rental cost $r_1 h_1$).

The general equilibrium solution under the developer specification in either (29a) or (29b) with national human capital markets is given in the Appendix and conforms to the efficient outcome. At the city level, in both cases, the Henry George theorem applies ($\delta_1 W_1 = \tfrac{1}{2} b n_1^{1/2}$) and city sizes still satisfy equation (8). However, in (29a), T_1 is reduced by rh_1 (from $\tfrac{1}{2} b n_1^{1/2}$ in eq. [7]), so, in effect, workers pay for their human capital. In (29b), workers pay directly for their human capital and T_1 remains as in equation (7). Proposition 5 summarizes this discussion.

PROPOSITION 5. If developers can dictate h_i levels for their cities, local human capital spillovers potentially can be internalized, resulting in efficient contemporaneous market outcomes and growth paths.

Policy Issues

Why is implementation of proposition 5 problematical? Let us start with the specification in (29b), where individuals bear the rental costs of their human capital investments. In an efficient solution to (29b), developers not only must specify $\overset{*}{h}_i$ but must enforce it. The most critical problem is that $\overset{*}{h}_i$ is not ‘‘self-enforcing,’’ unlike the specification of city size, n_i. Recall that n_i is a free-mobility equilibrium, so developer-announced strategies (in a properly specified game, as in Helsley and Strange [1993]) are self-enforcing: in equilibrium, no developer has an incentive to alter n_i and no family has an incentive to move a member from one city to another. In contrast, $\overset{*}{h}_i$ is not self-enforcing. On the basis of private returns, ϕ_i, rather than social returns, a member of any city would prefer to invest an $h_i < \overset{*}{h}_i$. This means that developers must monitor and enforce the educational choices of every entrant. Not only would such a pedigree requirement be legally problematical in a democratic society (unlike effective specification of n_i through zoning), but in reality h_i is not readily quantified (again, unlike n_i), given on-the-job training and vast quality differences across years of formal education. Moreover, even if $\overset{*}{h}_i$ were monitorable, each developer, at least at any instant, has an incentive to cheat and allow in a marginal member, j, with $h_{ij} < \overset{*}{h}_i$. The last ($j$th) entrant gets the high social returns if all other entrants have $\overset{*}{h}_i$ for a lower private cost, rh_{ij}. Such an entrant could bribe, with his surplus, the otherwise zero-profit developer to let him in. Finally, and related to these problems of observability and enforceability, in order for individuals to undertake $\overset{*}{h}_i$, they must trust that there will be an equilibrium solution in which developers can and will each universally enforce $\overset{*}{h}_i$ in their cities. This comment is highlighted in an overlapping generations context (see below), where discrete time separates investment decisions and realization of investment outcomes.

Having developers, rather than residents, invest (at least at the margin) in human capital of residents, as in equation (29a), solves the incentive problem in enforcement of $\overset{*}{h}_i$. Since developers bear the costs, they need full productive outcomes to recover their costs in the specification of T_i payments. However, every developer has an incentive to steal others' investees: hiring outsiders with already invested $\overset{*}{h}_i$'s, requiring no payments (lower T's) for higher h_i's from them. To recover costs of their investments, developers must be able to track down (former) residents who leave with this capital and force repayment. This problem brings back into focus the no-slavery constraint in implementing human capital contracts. How can developers, by themselves, enforce repayment of human capital loans with no collateral? Again, in an overlapping generations context, the problem is highlighted by asking how individual cities can recover the marginal social benefits of investing in their former children when children are free to migrate as adults, the traditional brain-drain problem.

The inability of land developers or autonomous local governments to implement optimal human capital investments implies a role for state and national governments in educational policy, beyond ensuring minimal educational standards in a democracy. In most countries, state and national governments have a major role in education. In the United States, while local governments have administrative responsibility for public schooling, federal and mostly state governments provide a substantial portion of funding for public schooling (63 percent in 1993). Moreover, the federal and state governments have a strong role in higher education. Historically, under the 1862 Morill Act, the federal government distributed thousands of acres of land to the states for the purpose of providing ‘‘a liberal and practical education.’’ Today the federal government’s role through student loan programs remains critical, but the states bear major responsibility for public higher education.

All this suggests that federal and state policy could work to move us in the direction of more efficient human capital investment levels. The notion developed in this paper that deficiencies in investment are specific to occupation and industry would suggest that, beyond localities, states that are better informed than the national government about their own needs and changing industrial composition could play the stronger role, as they do. On the other hand, states, themselves, face brain-drain and incentive issues.

IV. Conclusions and Extensions

In the Introduction we stressed two themes for this paper: the effect of urbanization on growth efficiency and the effect of growth on urbanization. With respect to the second, we demonstrated the strong positive relationship between city sizes and local educational attainment. We developed a model of parallel growth in sizes and numbers of cities, with economic and population growth. In terms of the effect of urbanization on growth, we argued that, in theory, urban institutions could lead to efficient growth with the internalization of local knowledge spillovers, but implementation faced significant problems. We also commented on the effect of urbanization on inequality. In our model there is measured nominal and real income inequality across cities, although not consumption differences.

In terms of extensions, our continuous-time dynastic formulation does not allow for much flexibility in the specification of how human capital is transmitted across generations. Reformulating the problem using a discrete-time overlapping generations framework allows alternative specifications of the intergenerational transmission of human capital to be explored (Black 1998). When parents have a ‘‘joy of giving’’ type bequest motive (e.g., Galor and Zeira 1993), our basic results hold. Using bequests, when young, workers decide how much to invest in human capital in the city they choose to locate in, ensuring equalized utility levels across cities. When old, they allocate income earned between consumption and bequests. If liquidity constraints are nonbinding, the results in equation (16) are duplicated.

Other specifications of intergenerational transmission of human capital can lead to persistent inequalities in utilities across city types, starting from initially identical populations (Black 1998). For example, we can combine the altruistic joy of giving bequest motive with parental or peer group effects, following Benabou (1996) and Durlauf (1996). While an initial generation allocates itself across city types to equalize utility, suppose that subsequent generations' cost of human capital is reduced according to the educational level in the city in which they reside, only if they stay in their parents' city. Given this, human capital costs and levels will diverge by city type from initial levels and so will consumption levels or real incomes net of human capital costs. Persistent inequality can also arise under a Becker, Murphy, and Tamura (1990) specification in which parents care about the ''quality'' of their children and make investment choices for their children.

Appendix

Growth Paths

Steady-State Levels

Full details of the dynamics of the model are given in Black and Henderson (1997). The basic dynamics turn out to be standard. For a phase diagram in h, c space, from equations (23) and (24), we can show that the $\dot{c} = 0$ locus is a vertical line. The $\dot{h} = 0$ locus is an inverted U with a maximum to the right of the steady-state $\bar{h}$ in (27). The motion in the system gives a stable arm leading to $\bar{h}$, $\bar{c}$. The steady state lies below the locus of critical values of c given by the $\dot{H} \geq 0$ constraint, as does generally the stable arm. Transversality is satisfied at $\bar{h}$, $\bar{c}$, for $\rho > g$, as assumed. To rule out paths other than the stable arm, we prove that other potential paths violate transversality.

Steady-State Growth

For steady-state growth, there are no transition dynamics: the economy is always at the steady-state growth rate. What is that growth rate? For consumption per person, consumer optimization implies $c(t) = c_0 \exp [(A - \rho)/\sigma]t$. In Black and Henderson (1997), we prove that the solution to (24) for $h(t)$ requires $h(t)$ to grow at the rate $(A - \rho)/\sigma$ as in equation (25), if transversality is to be satisfied, which also requires equation (26b) to hold.

Proposition 4

For the planner's problem, given $m_1 n_1 = ze^{gt}$, total $\bar{X}_1$ for use in production of X_2 is $[zD_1 n_1^{\delta_1} h_1^{\psi_1+\theta_1} - zbn_1^{1/2} - (1 - z)bn_2^{1/2}]e^{gt}$, where the first term is total X_1 output (eq. [3]) and the next two are national commuting costs. Then total output of X_2 equals $m_2 n_2 D_2 n_2^{\delta_2} h_2^{\psi_2+\theta_2}(X_1 m_2^{-1} n_2^{-1})^{1-\alpha}$, where $m_2 n_2 = (1 - z)e^{gt}$. The planner's problem is then

$$\max_{n_2,n_1,h_2,h_1} \mathcal{L} = \frac{c^{1-\sigma}-1}{1-\sigma}e^{-(\rho-g)t} + \lambda_1 e^{gt}\{(1-z)^{\alpha}D_2 n_2^{\delta_2}h_2^{\psi_2+\theta_2} \times [zD_1 n_1^{\delta_1}h_1^{\psi_1+\theta_1} - zbn_1^{1/2} - (1-z)bn_2^{1/2}]^{1-\alpha} - c\} + \lambda_2[H - ze^{gt}h_1 - (1-z)e^{gt}h_2].$$

The solution to this problem is stated below. An alternative way to proceed is to have a quasi social planner, who allows national output markets to operate but controls h_1, h_2, n_1, n_2, and z, recognizing the effects on P. In this case, to solve for an efficient outcome, we substitute I_1 and I_2 from (9) and (13) into the representative family's dynamic optimization problem in (1). We then substitute in

$$P = \left[\frac{Q_1(\alpha-2\delta_2)h_1^{\epsilon_1}z}{Q_2(1-\alpha+2\delta_2)h_2^{\epsilon_2}(1-z)}\right]^{\alpha-2\delta_2},$$

obtained by combining (17) with (9), (13), $e^{gt}z = m_1 n_1$, and $e^{gt}(1-z) = m_2 n_2$. Optimizing with respect to h_1, h_2, c, and z yields with rearrangement

$$\frac{\overset{*}{h}_1}{\overset{*}{h}_2} = \frac{\epsilon_1(1-\epsilon_2)}{\epsilon_2(1-\epsilon_1)},$$

$$\overset{*}{h}_i = \frac{\epsilon_i}{1-\epsilon_i}\overset{*}{K}h,$$

$$\overset{*}{z} = \frac{(1-\epsilon_1)(1-\alpha+2\delta_2)}{(1-\epsilon_1)(1-\alpha+2\delta_2)+(1-\epsilon_2)(\alpha-2\delta_2)},$$

$$\frac{\overset{*}{I}_1}{\overset{*}{I}_2} = \frac{1-\epsilon_2}{1-\epsilon_1},$$

$$\overset{*}{K} = \frac{(1-\epsilon_1)(1-\alpha+2\delta_2)+(1-\epsilon_2)(\alpha-2\delta_2)}{\epsilon_1(1-\alpha+2\delta_2)+\epsilon_2(\alpha-2\delta_2)},$$

$$\overset{*}{P} = \overset{*}{Q}h^{(\epsilon_1-\epsilon_2)(\alpha-2\delta_2)},$$

$$\overset{*}{Q} = \left[\frac{\epsilon_1 Q_1\left(\dfrac{\epsilon_1}{1-\epsilon_1}\right)^{\epsilon_1-1}}{\epsilon_2 Q_2\left(\dfrac{\epsilon_2}{1-\epsilon_2}\right)^{\epsilon_2-1}}\overset{*}{K}^{\epsilon_1-\epsilon_2}\right]^{\alpha-2\delta_2}.$$

These equations, along with (17)–(20), give us the h_1/h_2, $\overset{*}{h}_1/\overset{*}{h}_2$, I_1/I_2, and $\overset{*}{I}_1/\overset{*}{I}_2$ comparison in proposition 4 in the text, by inspection. By substitution of the new values of $\overset{*}{h}_i$ into (8), (12), and (18), we can compare $\overset{*}{n}_1/\overset{*}{n}_2$ with n_1/n_2 and corresponding $\overset{*}{m}_1/\overset{*}{m}_2$ with m_1/m_2 to obtain detailed expressions.

For the rest of proposition 4, we combine first-order conditions corresponding to (15a) and (15e) with the new equation of motion to solve for steady-state levels.

For steady-state levels, when $\epsilon < 1$, we have

$$\overset{*}{\bar{h}} = \left(\frac{\overset{*}{A}}{\rho}\right)^{1/(1-\epsilon)}, \tag{A1}$$

$$\overset{*}{\bar{c}} = \left(\frac{\overset{*}{A}}{\rho}\right)^{1/(1-\epsilon)} \left(\frac{\overset{*}{B}\rho}{\overset{*}{A}} - g\right), \tag{A2}$$

$$\overset{*}{B} \equiv \overset{*}{A}\overset{*}{z}(1 - \alpha + 2\delta_2)^{-1}(1 - \epsilon_1)^{-1}\overset{*}{K}.$$

When the equilibrium and the optimum are compared, $\overset{*}{\bar{h}} > \bar{h}$ and $\overset{*}{\bar{c}} > \bar{c}$ iff $\overset{*}{A} > A$. In Black and Henderson (1997), we prove $\overset{*}{A} > A$ if $\epsilon_1 \geq \phi_1$ and $\epsilon_2 \geq \phi_2$, with strict inequality for one. That is, $\overset{*}{A} > A$ if there are human capital spillovers.

Proposition 5

Assume, for example, that h_1 and h_2 are chosen by developers, who borrow in a national human capital market from capital owned by families, at a prevailing rental rate of r. In city type 1, the developer's problem is (29a) in the text, or

$$\max \tfrac{1}{2} bn_1^{3/2} - T_1 n_1 - rh_1 n_1 + \lambda(D_1 n_1^{\delta_1} h_1^{\psi_1+\theta_1} + T_1 - \tfrac{3}{2} bn_1^{1/2} - I_1). \tag{A3}$$

Here in equilibrium, $T_1 = D_1\delta_1 n_1^{\delta_1} h_1^{\psi_1+\theta_1} - rh_1 = \tfrac{1}{2} bn_1^{1/2} - rh_1$; so while the Henry George theorem still holds, T_1 is reduced by implied capital rental payments. Solving the problem for city types 1 and 2, we get the text expressions for n_1 and n_2 and

$$r = \epsilon_1 Q_1 h_1^{\epsilon_1 - 1} = \epsilon_2 Q_2 P^{1/(\alpha-2\delta_2)} h_2^{\epsilon_2 - 1} \tag{A4}$$

and

$$I_1 = Q_1 h_1^{\epsilon_1}(1 - \epsilon_1) = I_2 = Q_2 P^{1/(\alpha-2\delta_2)} h_2^{\epsilon_2}(1 - \epsilon_2). \tag{A5}$$

Note that since capital is here a freely mobile input, chosen by developers, labor incomes in national markets must be equalized for families in allocating members. Now, in the family's optimization problem,

$$\dot{H} = I_1 P^{-1} z e^{gt} + I_2 P^{-1}(1 - z)e^{gt} + P^{-1} rh e^{gt} - ce^{gt},$$

where capital rent is paid in units of X_1 but new additions to capital come from X_2. In choosing z, in the family's problem to maximize

$$\frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho-g)t}$$

subject to (just) the equation of motion, we get $I_1 = I_2$. (Note now that I's are adjusted for T's reflecting capital rental payments.)

Besides (A4) and (A5), we substitute into $m_1X_1 = m_2n_2x_1 + m_1(bn_1^{3/2}) + m_2(bn_2^{3/2})$ for m_1n_1 $(= ze^{gt})$, m_2n_2 $(= [1 - z]e^{gt})$, X_1, x_1, and P (from [A5]) to get $zh_1^{\epsilon_1}Q_1 = (1 - z)Q_2P^{1/(\alpha-2\delta_2)}h_2^{\epsilon_2}(1 - \alpha + 2\delta_2)/(\alpha - 2\delta_2)$. Combining this, (A4), (A5), and $h = zh_1 + (1 - z)h_2$ gives the efficient outcomes listed above for $\overset{*}{h}_1/\overset{*}{h}_2$, and $\overset{*}{z}$. Solving the family's optimization problem gives (A1)–(A3).

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