TUESDAY 13th APRIL 2010

Advances in Spatial Analysis & e-Social Science

# The Dynamics of Skyscrapers

## Scaling, Allometry and Sustainability

Michael Batty

University College London

m.batty@ucl.ac.uk

http://www.casa.ucl.ac.uk/



## Outline

- Defining Skyscrapers
- Defining Scaling: Competition In Cities
- London and Hong Kong: Baseline Exemplars
- The Top World Cities
- The World's Buildings
- Glimpses of Allometry
- Dynamics of Skyscraper Heights: Rank Clocks
- Next Steps ….. A Different Data Source

## Defining Skyscrapers

Tall structures whose height is much larger than their building footprint and are qualitatively different in construction from smaller ones

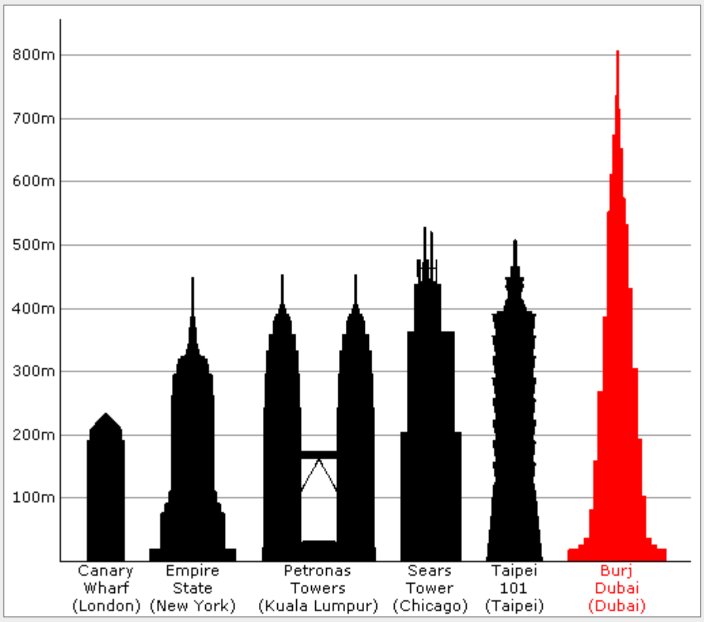
The steel frame, the elevator, the telephone, and the revolving door are all key to their invention

They appeared the first time in late 19th century Chicago: Louis Sullivan, Daniel Burnham, …




Chicago's Monadnock Building is the proto-typical skyscraper built in 1891; 16 stories, 60 m

Currently the world's tallest building is the Burj Dubai standing at 818 m or 160 stories (habitable floors)






The conventional wisdom is that we define a tall building as being greater than 30 metres or maybe greater than 8, 10 or 12 stories

In fact, buildings greater than 30 metres and less than 100 metres are "high rise" while buildings greater than 100 metres are "skyscrapers"

The average height of 'stories' over all high buildings is lowest in Paris at 3.27 m and largest in Dubai at 4.32 m

## Defining Scaling: Competition In Cities

The ordering of elements such that there are a very small number of large objects/events and large number of small reflects 'competition'.

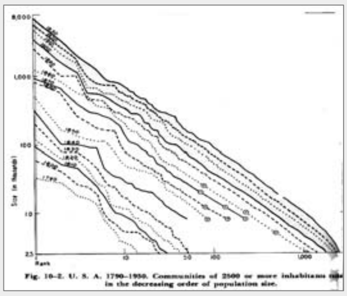
Such systems are asymmetric in that, often but not always, to be 'big' one must be 'small' first. The upper or fat tail of such distributions can be approximated by a power law

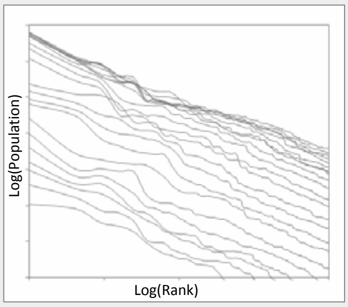


In human systems, such growth behaviour depends on competition for scarce resources and in cities, this is called 'agglomeration' (so-called positive economies of scale).

Many simple models of how such scaling occurs have been proposed based on laws of proportionate effect which lead to lognormal-like distributions whose fat tail can often be approximated as one of scaling, by power laws.

The archetypal example of such scaling is for entire city populations whose distributions can be approximated by a transformation of the frequency called the ‘rank size distribution’, popularised by Zipf (1949)








There is considerable debate (and semantic confusion) about the nature of the competitive forces and the shape of the tails but for skyscrapers, there are interesting differences from other competitive phenomena

*First*, few have been destroyed – i.e. there is only ‘growth’ of new buildings; *second*, high-rise buildings are ‘qualitatively’ different from small; and *third*, buildings do not actually grow.

Centre for Advanced Spatial Analysis UCL

This is a debatable point. Yesterday when I gave a talk on large cities at the British Museum, Scott Branting from Chicago pointed out that there are actually some buildings that have growth built into them.

The Blue Cross Blue Shield Building in Chicago was completed at 33 stories in 1997 with another 24 stories planned for later; and later has now come to pass in 2010



***CityPoint*** was built in 1967 as a 35-storey, 122 metres (400 ft) tall headquarters for British Petroleum (now BP) and was originally named Britannic House. It was refurbished in 2000 which increased the height to 127 metres (417 ft) and increased the available floor space.

Centre for Advanced Spatial Analysis

That there is competition for building ever higher both within and between cities there is little doubt – examples in New York City are key

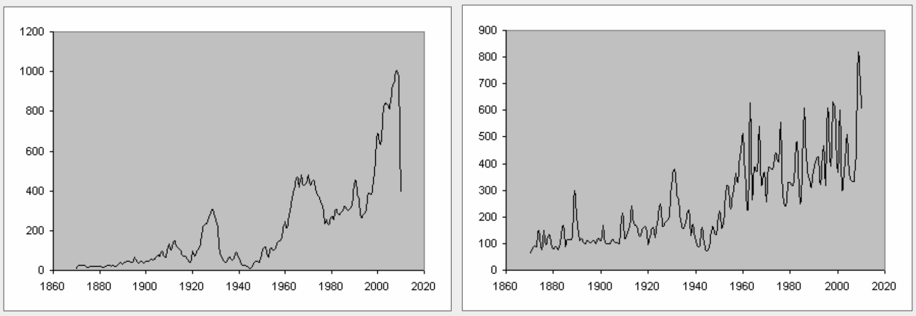
*The Chrysler Building, an Art Deco skyscraper stands at 319 metres; it was the world's tallest building for 11 months before it was surpassed by the Empire State Building in 1931. After the destruction of the World Trade Center, it was again the second-tallest building in New York City until December 2007, when the spire was raised on the 365.8-metre (1,200 ft) Bank of America Tower, pushing the Chrysler Building into third position. In addition, The New York Times Building which opened in 2007, is exactly level with the Chrysler Building in height.[6]*



## Two Digressions on Competition

Scaling, power laws and size distributions is all about competition and skyscrapers are one the great exemplars of capitalism. It is said that the Reverend S. Parkes Cadman dedicated the Woolworth Building as a "cathedral of commerce" at its official opening on April 23, 1913.

Such buildings tend to become ever higher during times of irrational exuberance. Here is the distribution of the top 29000 high buildings in the world from 1876




Frequency of buildings > 30 m (left) and highest building constructed by year since 1870s(right)

Centre for Advanced Spatial Analysis UCL

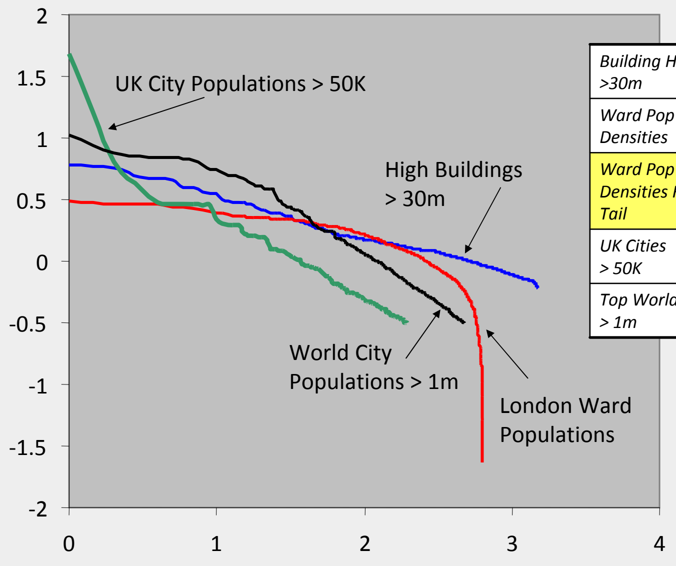
And here is an recent extreme example of competition within a world city: Hong Kong


Centre for Advanced Spatial Analysis UCL

It is easy to guess that high buildings follow a scaling law, but how does this competition compare to other urban distributions such as population densities in cities, populations of different cities, and so on.

To begin, let us look at some distributions of these entities for London, for the UK and for the world

Centre for Advanced Spatial Analysis UCL




| | | | |
|---|---|---|---|
| *Building Heights >30m* | 1486 | 0.99 | **-0.31** |
| *Ward Pop Densities* | 633 | 0.64 | **-0.57** |
| *Ward Pop Densities Fat Tail* | 160 | 0.89 | **-0.20** |
| *UK Cities > 50K* | 194 | 0.97 | **-0.70** |
| *Top World Cities > 1m* | 478 | 0.98 | **-0.73** |

*Intra-city or intra-urban distributions tend to be flatter, less extreme, than inter-city or inter-urban*

Centre for Advanced Spatial Analysis UCL

# London and Hong Kong: Baseline Exemplars

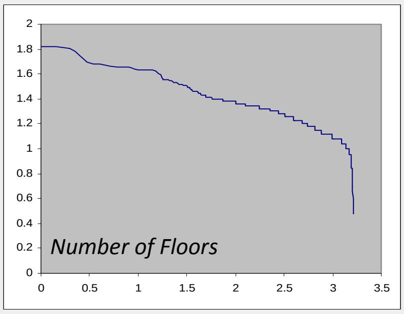
The ***Emporis*** Database: data on high rise buildings > 30 m for many cities, e.g. 8 in UK,

340,000 buildings world-wide with height, stories, floor area, land use type, year of build,

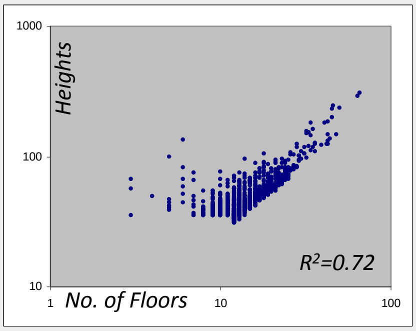
Many of these data fields are missing so a much reduced set is only usable for each city; e.g. London has 2495, but 1598 have height data.

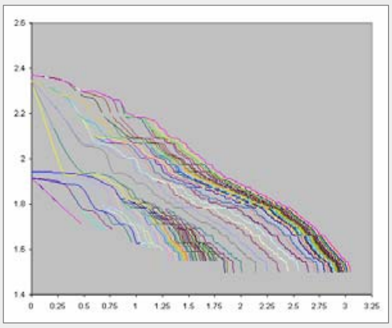


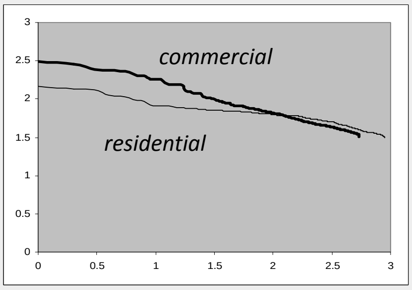
We will look first at three distributions for each city: the scaling of height and number of stories, the prediction of height from stories, and change in scaling from the late $19^{th}$ C


Centre for Advanced Spatial Analysis UCL



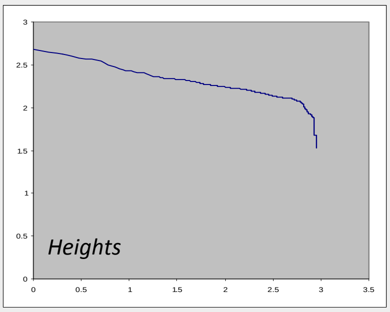



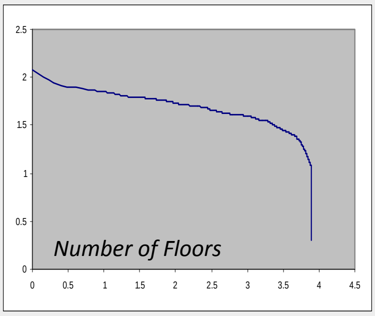



| Type | Slope | $r^2$ |
|---|---|---|
| All heights | -0.3169 | 0.9875 |
| Heights from floor stories | -0.3186 | 0.8764 |
| All floors | -0.3248 | 0.8433 |
| Heights com+mixed | -0.3792 | 0.9945 |
| Heights residential | -0.2428 | 0.9232 |

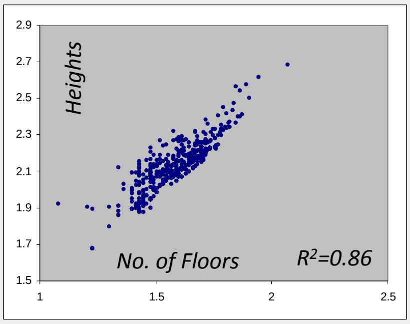
Centre for Advanced Spatial Analysis UCL

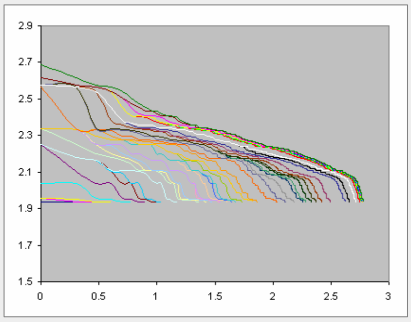
We can do the same for Hong Kong, our other exemplar, and we will simply show the heights scaling for now, and then for the rest of the cities, simply the results

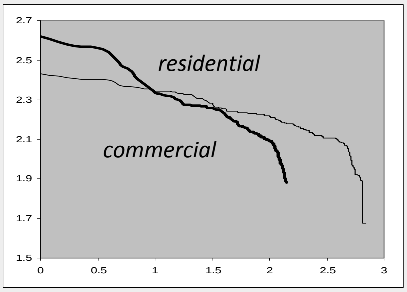



Centre for Advanced Spatial Analysis UCL






| Type | Slope | $r^2$ |
|---|---|---|
| All heights | -0.3046 | 0.6999 |
| Heights from floor stories | -0.2686 | 0.7145 |
| All floors | -0.2973 | 0.8018 |
| Heights com+mixed | -0.2779 | 0.9804 |
| Heights residential | -0.1679 | 0.9703 |

Centre for Advanced Spatial Analysis UCL

## The Top World Cities

We have taken the top 50 cities in terms of population starting with Tokyo (28 million) down to Melbourne (3 million)

Only 38 have good enough data, and thus we have selected these plus three other iconic cities – Dubai, Barcelona, Kuala Lumpur that have unusual high buildings

Centre for Advanced Spatial Analysis UCL

| | |
|---|---|
| Tokyo, Japan - 28,025,000 - 3 478 | Santiago, Chile - 5,261,000 - 1587 |
| Mexico City, Mexico - 18,131,000 - 1637 | Guangzhou, China - 5,162,000 - 603 |
| Mumbai, India - 18,042,000 - 1366 | St. Petersburg, Russian Fed. - 5,132,000 - 962 |
| Sáo Paulo, Brazil - 17, 711,000 - 6850 | Toronto, Canada - 4,657,000 - 2883 |
| New York City, USA - 16,626,000 -78 523 | Philadelphia, USA - 4,398,000 - 703 |
| Shanghai, China - 14,173,000 – 1222 | Milano, Italy - 4,251,000 - 747 |
| Los Angeles, USA - 13,129,000 - 1771 | Madrid, Spain - 4,072,000 - 1429 |
| Calcutta, India - 12,900,000- 527 | San Francisco, USA - 4,051,000 - 1230 |
| Buenos Aires, Argentina - 12,431,000 - 1893 | Washington DC, USA - 3,927,000 - 1402 |
| Seóul, South Korea - 12,215,000 - 3099 | Houston, USA - 3,918,000 - 3292 |
| Beijing, China - 12,033,000 - 1122 | Detroit, USA - 3,785,000 - 696 |
| Õsaka, Japan - 10,609,000 - 1326 | Frankfurt, Germany - 3,700,000 - 6632 |
| Rio de Janeiro, Brazil - 10,556,000 - 3042 | Sydney, Australia - 3,665,000 - 1190 |
| Jakarta, Indonesia - 9,815,000 - 837 | Singapore, Singapore - 3,587,000 - 6801 |
| Paris, France - 9,638,000 - 971 | Montréal, Canada - 3,401,000 - 550 |
| Istanbul, Turkey - 9,413,000 - 2553 | Berlin, Germany - 3,337,000 - 1125 |
| Moscow, Russian Fed. - 9,299,000 - 2330 | Melbourne, Australia - 3,188,000 – 723 |
| London, United Kingdom - 7,640,000 - 2507 | *Barcelona – 716 – 1605602* |
| Bangkok, Thailand - 7,221,000 - 949 | *Dubhai 1175 – 1241000* |
| Chicago, USA - 6,945,000 - 2761 | *Kuala Lumpur – 766 - 1 800 674* |
| Hong Kong, China - 6,097,000 - 8086 | |



| | Av St | Slope | $r^2$ | | Av St | Slope | $r^2$ |
|---|---|---|---|---|---|---|---|
| LA | 4.1940 | -0.6773 | 0.8197 | Barcelona | 3.9358 | -0.4431 | 0.8993 |
| Frankfurt | 3.8447 | -0.6135 | 0.8768 | Sao Paolo | 3.7728 | -0.4429 | 0.7170 |
| Houston | 4.1404 | -0.5884 | 0.7849 | Sydney | 3.8094 | -0.4348 | 0.9561 |
| Moscow | 4.0626 | -0.5654 | 0.8380 | Beijing | 3.9287 | -0.4290 | 0.7999 |
| San Francisco | 4.0334 | -0.5649 | 0.8409 | Mumbai | 3.5377 | -0.4262 | 0.9250 |
| Madrid | 3.9703 | -0.5469 | 0.9293 | Shanghai | 4.2147 | -0.4122 | 0.8598 |
| Detroit | 4.0277 | -0.5448 | 0.9203 | Buenos-Aires | 3.5224 | -0.4110 | 0.6520 |
| Toronto | 3.4009 | -0.5195 | 0.8406 | Tokyo | 4.1029 | -0.4033 | 0.8271 |
| Philadelphia | 3.9459 | -0.5156 | 0.8608 | Mexico-City | 3.9656 | -0.3863 | 0.7881 |
| Singapore | 3.5658 | -0.5128 | 0.8129 | Santiago | 3.5315 | -0.3834 | 0.8000 |
| St Petersburg | 4.0417 | -0.5078 | 0.7607 | Seoul | 3.9230 | -0.3825 | 0.8550 |
| All Cities-World | 3.6714 | -0.4874 | 0.8898 | Istanbul | 4.0284 | -0.3264 | 0.7179 |
| Chicago | 3.5154 | -0.4856 | 0.7909 | Milano | 3.3808 | -0.3225 | 0.9796 |
| Dubai | 4.3194 | -0.4786 | 0.8273 | Jakarta | 3.8144 | -0.3177 | 0.7146 |
| New York | 3.4649 | -0.4750 | 0.9305 | Washington | 4.0746 | -0.3153 | 0.9145 |
| Melbourne | 3.7390 | -0.4735 | 0.9006 | Rio de Janeiro | 3.3043 | -0.3122 | 0.9107 |
| Paris | 3.2732 | -0.4724 | 0.5742 | Bangkok | 3.7959 | -0.3024 | 0.7780 |
| Guangzhou | 4.0083 | -0.4678 | 0.8447 | Calcutta | 3.4231 | -0.2715 | 0.6600 |
| Montreal | 3.9423 | -0.4625 | 0.7441 | Osaka | 4.1274 | -0.2679 | 0.8348 |
| Berlin | 3.4974 | -0.4514 | 0.8737 | KL | 4.2134 | -0.4492 | 0.9195 |

*London and HK are not in this list yet ….*

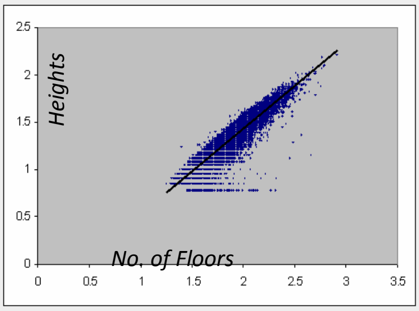
## The World's Buildings

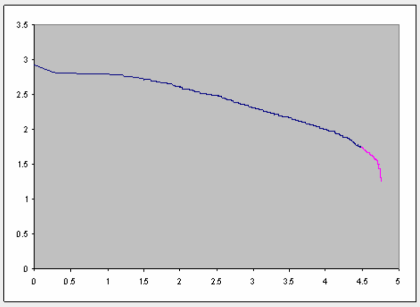
We can of course aggregate the data we have looked at into all buildings and we have done this – there are 57000 usable heights from 340K buildings giving you a crude idea of the accuracy and error in this data set.

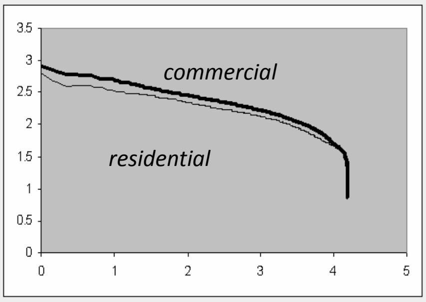
There are 33314 usable stories which is less than heights

Centre for Advanced Spatial Analysis

UCL






*We have not done the temporal scaling relations as yet*

Centre for Advanced Spatial Analysis

UCL

| *Regressions* | Min | Av | No | Intercept | Slope | $r^2$ |
|---|---|---|---|---|---|---|
| *All buildings* | 3.6714 | | | | | |
| All heights | 18 | 70 | 56999 | 3.8932 | -0.4874 | 0.8898 |
| All heights less long tail | 72 | 113 | 21053 | 3.3029 | -0.3290 | 0.9906 |
| All heights from floors stories | 4 | 72 | 33314 | 3.8533 | -0.5043 | 0.7626 |
| As above less long tail | 73 | 117 | 11850 | 3.0857 | -0.2849 | 0.9398 |
| Heights com+mixed | 13 | 81 | 15464 | 3.8037 | -0.5240 | 0.8546 |
| Heights residential | 12 | 66 | 16075 | 3.4930 | -0.4581 | 0.9081 |
| Heights viz floorarea | | | 8299 | 0.2000 | *0.3768* | 0.4222 |
| Height rank | | | 3445 | 3.1134 | *-0.3303* | 0.9611 |
| Floor rank | | | 4218 | 6.5103 | *-0.5641* | 0.9511 |



# Dynamics of Skyscraper Heights: Rank Clocks

I am going to digress once again because there is another strand to all of this and this relates to asking what happens to the set of all skyscraper heights as new skyscrapers are built through time

To explore this we look at the frequency of heights in the form of scaling laws

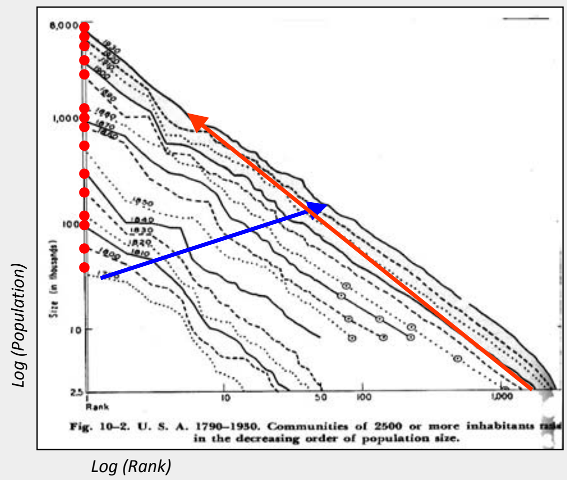
We measure this frequency by ranking the cities or buildings from largest to smallest and plotting this rank size – let us show this for the US populations from 1790 to date for all cities

What we will see is <u>great stability</u> in the rank size – almost perfect power laws in the upper tail as we have seen for high buildings in London

But <u>massive volatility</u> in how these ranks change through time

Centre for Advanced Spatial Analysis UCL

## The Key Issue: Macro Stability & Micro Volatility



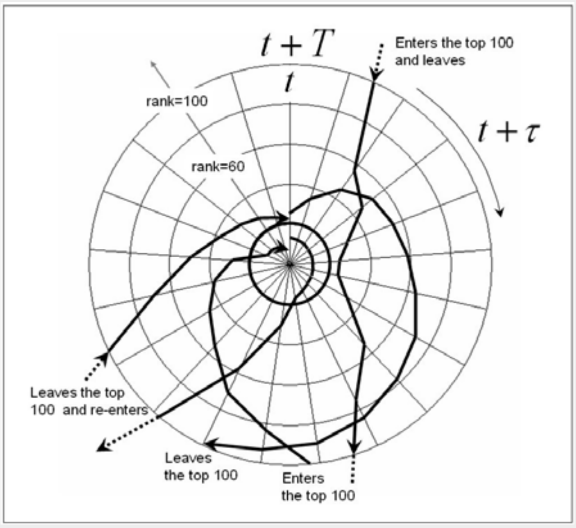

- New York

Houston, TX →

Richmond, VA →

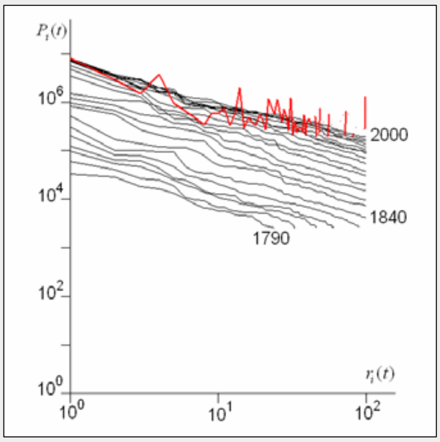
From George Kingsley Zipf (1949) ***Human Behavior and the Principle of Least Effort*** (Addison-Wesley, Cambridge, MA)

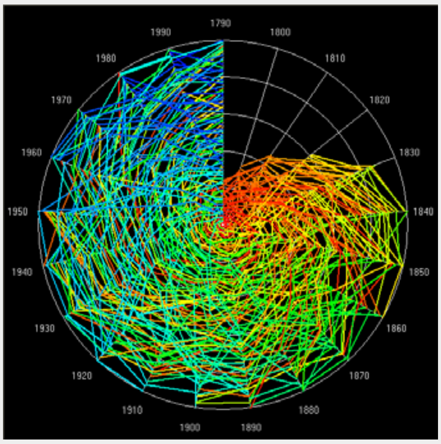
Centre for Advanced Spatial Analysis UCL




The Idea of a Rank Clock –rank is from number 1 at centre to 100 at edge and time goes in years in the usual clockwise direction

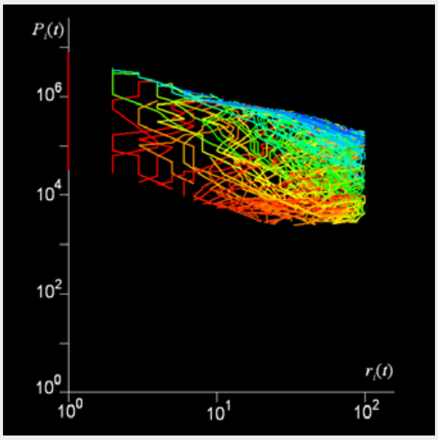


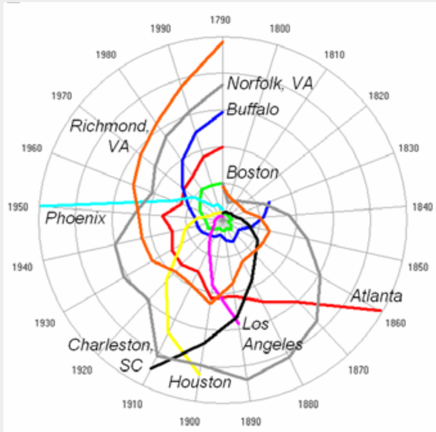



My point will be that the 'morphology' of the clock should tell us something – i.e. the increase in cities, the volatility of ranks and so on.

Centre for Advanced Spatial Analysis
UCL

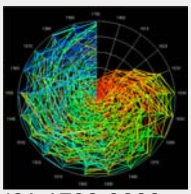


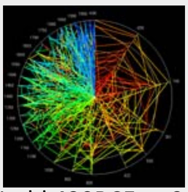



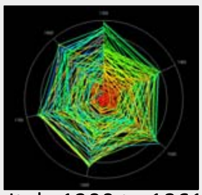

The rudimentary software for this in on our web site at
[http://www.casa.ucl.ac.uk/software/rank.asp](http://www.casa.ucl.ac.uk/software/rank.asp)

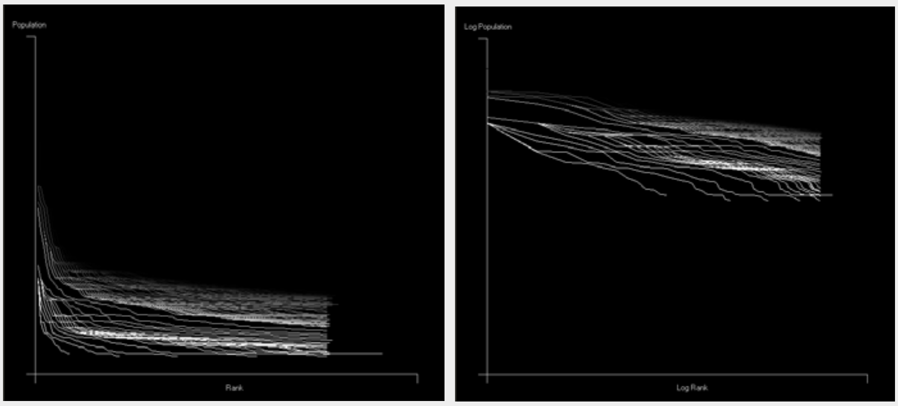


In fact, these clocks are difficult to read as static pictures – like all clocks they imply a dynamic that we need to explore and to do these we need to look at different trajectories in the clocks. We have animated some of these & I will show some by way of digression.


USA 1790-2000

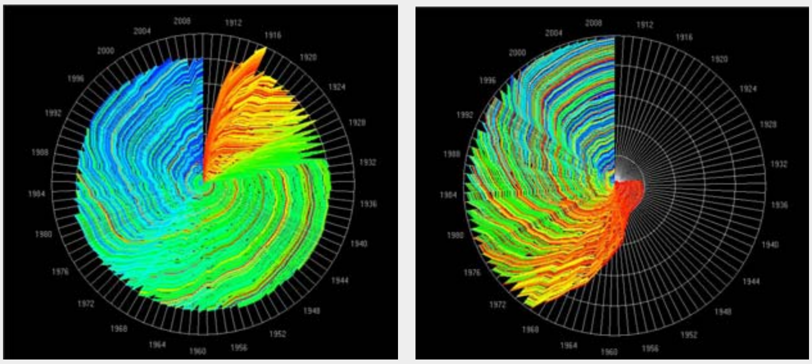

World 430BCE to 2000


Italy 1300 to 1861



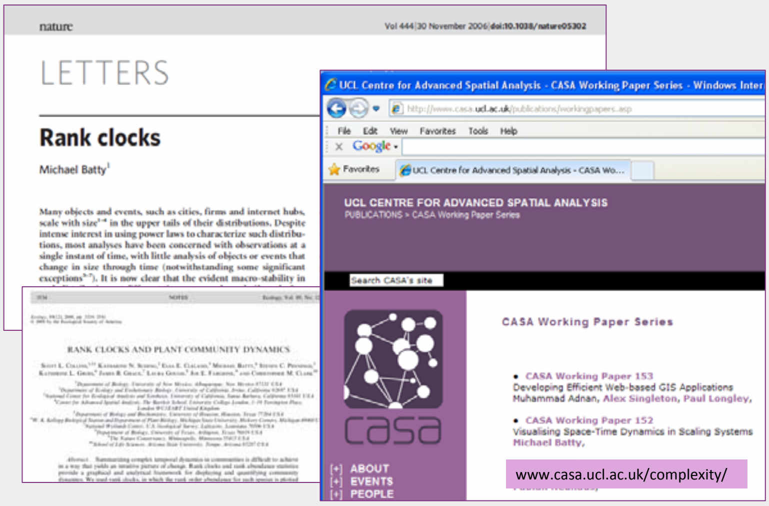

Rank Size Relations for the Top 100 High Buildings in the New York City from 1909 until 2010
*power form (left)* *log form (right)*





Rank Clocks of the Top 100 High Buildings in the New York City (a) and the World (b) from 1909 until 2010
*There is much more work to do on all this and I am only giving you a taste of this, now back to shape and size*

Some references before I move to my last bit of the talk






## Glimpses of Allometry

I am fast running out of time but my ultimate purpose is to ask the fundamental question

*"How Big is a City?" & "How Sustainable is It?"*

This is a question that I do not believe can be answered any other way, in terms of population for example; it must be looked at with respect to how we use space and this relates to the size of buildings

From the data base, we have floor area; we do not have volume or surface area so we cannot get any detailed sense of how a building's volume changes as it gets bigger

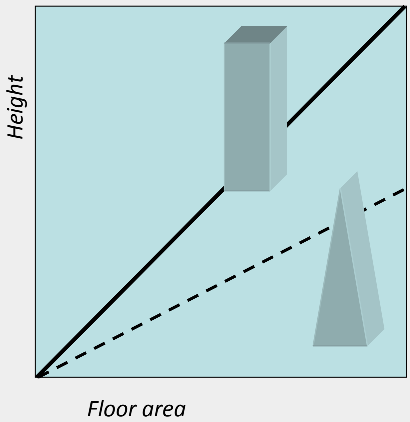
Now this is not a talk on allometry per se but as a building gets larger in volume, then its surface area must increase faster than the usual Euclidean relation for buildings require access to natural light



It has been shown that for building volumes, the building surface area scales as the power of ¾ of the volume, following Kleiber's Law, not as 2/3 of the volume as the geometric relation would suggest.

We have done quite a bit of work on this and I will refer to the papers later, but all we have from this data set is floor area and we might suppose that floor area scales as height.

Centre for Advanced Spatial Analysis UCL

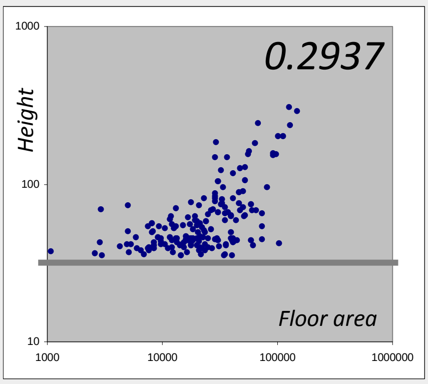



These are the simplest theoretical relations but there are other shapes with plinths and so on …

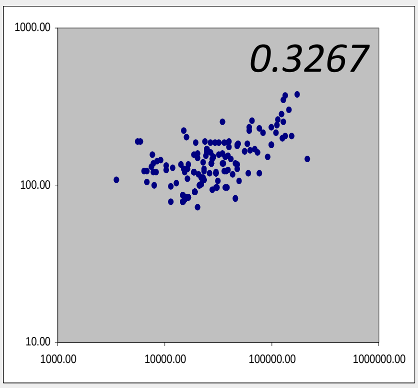
We can now quickly look at our data bases and these relations



Here we look at the relationship between Height and Usable (not Gross) Floor Area for London, HK, and the World's Buildings



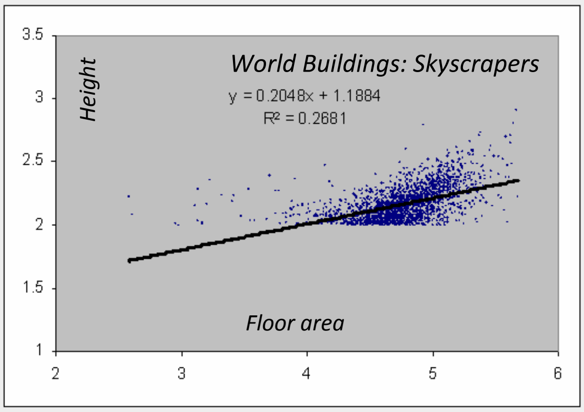

London




Hong Kong

Centre for Advanced Spatial Analysis UCL

From this it looks like height scales more as $H^3 \sim F$ than $H^1 \sim F$




*0.3259; for skyscrapers above, this is 0.2048*



## Next Steps: A Different Data Source

We have good data from LIDAR and vector building footprint and we are working hard to examine all the relations in this paper using our geometric database of London which has currently 3.6 million building blocks from which we can get surface area, volume etc.

Note that the problems of defining a building …

Eur. Phys. J. B 63, 303–314 (2008)
DOI: 10.1140/epjb/e2008-00251-5

THE EUROPEAN PHYSICAL JOURNAL B

## Scaling and allometry in the building geometries of Greater London

M. Batty[1,a], R. Carvalho[2,b], A. Hudson-Smith[1,c], R. Milton[1], D. Smith[1], and P. Steadman[3]

[1] Centre for Advanced Spatial Analysis, University College London, 1-19 Torring
[2] School of Mathematical Sciences, Queen Mary, University of London, Mile End
[3] Bartlett School of Architecture and Planning, University College London, Wat
London WC1E 6BT, UK

Received 7 December 2007 / Received in final form 15 April 2008
Published online 2 July 2008 – © EDP Sciences, Società Italiana di F

**Abstract.** Many aggregate distributions of urban activities such as c
any work exists on the properties of spatial distributions within indi
siderable knowledge about their fractal structure. We redress this here
in a world city using data on the geometric properties of individual
power laws can be used to approximate the size distributions of build
butions which have been widely studied as rank-size and lognormal d
*Behavior and the Principle of Least Effort* (Addison-Wesley, Cambridg
*Économiques* (Librarie du Recueil Sirey, Paris, 1931)]. We then extend
ships between buildings in terms of their different geometric size prope
analysis of building heights from the **Emporis** database which suggests
The data base for Greater London is then introduced from which we
scaling properties we explore. We examine key allometric relationships
illustrating how building shape changes according to size, and we exte
of buildings according to land use types. We conclude with an analysi
of building geometries which supports our non-spatial analysis of scali

**PACS.** 89.65.Lm Urban planning and construction – 89.75.Da Syste
Structures and organization in complex systems

BUILDING RESEARCH & INFORMATION (2009) 37(5-6), 455–467

Routledge

RESEARCH PAPER

## Wall area, volume and plan depth in the building stock

Philip Steadman[1], Stephen Evans[2] and Michael Batty[3]

[1]Bartlett School of Graduate Studies, [2]Centre for Advanced Spatial Analysis, and [3]Bartlett School of Planning, The Bartlett, University College London, 1–19 Torrington Place, London WC1E 6BT, UK
Emails: j.p.steadman@ucl.ac.uk, stephen.evans@ucl.ac.uk and m.batty@ucl.ac.uk

The relationship between the volume of a building and its wall area follows an allometric rule that implies that building shape distorts to capture as much surface area, hence natural light, as possible as it increases in size. For a sample of house plans, Bon in 1973 established that the relationship between wall area W and volume V scaled as $W \sim V^{0.77}$, and Steadman in 2006 demonstrated a similar relationship for his archetypal building. Empirical work in Cambridge and Swindon, UK, also revealed a similar allometry as measured by the depth ratio based on V/W, which provides a direct measure of the way building shapes become distorted with increasing size. This paper demonstrates positive allometry for building blocks taken from a large urban database (approximately 3.2 million blocks) for Greater London which is constructed from Ordnance Survey building footprint data augmented by remote sensing light detection and ranging (LIDAR) height data. For the domestic and then non-domestic stock, the blocks are categorized into eight bands and the depth ratios in six inner-London boroughs including the City, which is the financial quarter, are then examined. This is demonstrated in two ways – first, from the depth ratio; and second, from fitting allometric relationships to the band data. The allometric coefficients converge to values of around 0.77, thus confirming the magnitude of Bon's relationship, implying that positive allometry not only is a feature of small samples of houses and archetypal buildings, but also is more generally the case for real building databases at the very largest urban scales.

Keywords: allometry, building envelope, building geometry, building stock, built form, depth ratio, plan depth, surface area, volume



*We have a lot of things to do, apart from exploring the dynamics which is a separate strand in all this*

1. It appears that storey height increases with the year of construction & also the 'newer' the city
2. Floor area increases with height and we 'think' from our London work that surface area does indeed increase at the ¾ power of volume
3. Building higher requires more artificial light

Centre for Advanced Spatial Analysis
UCL

4. Essentially it looks like the higher we build, the more energy per unit volume, floor area, surface area we consume
5. We should be able to compute the difference between what might be required in artificial light & the actual light in each building block and use this as a measure of sustainability
6. We can add up these values for each city, determine how big it is and how much energy it uses



*If there is time,*
*I will answer any*

# Questions

*www.casa.ucl.ac.uk/sky.ppt*

Centre for Advanced Spatial Analysis UCL