Visualising Space-Time Dynamics: 
*Graphs and Maps, Plots and Clocks*

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**Outline**

- City-Size Distributions: Scaling and the Rank-Size Rule
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- The Rank Clock and Other Visual Mnemonics
- Classic Exemplars: US City Populations 1790-2000
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- Animations: Rank Clocks and Rank Space
- Next Steps
The key thesis – we need to visualise dynamic systems where the system appears stable at the macro level but volatile at the micro – there are two nice quotes to start

“I will [tell] the story as I go along of small cities no less than of great. Most of those which were great once are small today; and those which in my own lifetime have grown to greatness, were small enough in the old days”


But cities have a remarkable degree of regularity and stability with respect to their size ...

“The size distribution of cities in the United States is startlingly well described by a simple power law: the number of cities whose population exceeds P is proportional to 1/P. This simple regularity is puzzling; even more puzzling is the fact that it has apparently remained true for at least the past century.”

When we look at things like city sizes, we do not find a normal distribution but we find something that is like a power law or at least a lognormal.

I am going to spare you the algebra although it is easy and this is how we get the rank size distribution from the frequency power.
Now if we take logs – i.e. a simple transformation, this power law becomes a straight line in 2-d space and it is this form that we refer to as the rank-size rule.

\[ P(r) = \frac{1}{r^\alpha} \]

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**Macro-Stability & Micro-Volatility**

Essentially we can illustrate the stability of city size population by showing how this rank size curve changes over time.

It remains quite stable for the US from 1790 to 1930 and this is what Zipf, amongst others, discovered in the 1930s. And it prompted the Krugman quote. It is all in Zipf’s famous book. And Paul Krugman in the late 1990s also said that Zipf’s Law and Pareto’s Law before, are the only real examples of iron laws in the social sciences. Let us see what he meant.
The Rank Clock and Other Visual Mnemonics

The idea of the rank clock is to discard population because rank is its equivalent and just plot changes in rank with respect to time.

We can do this in rank space but it is not as effective as in polar coordinate space.

Here is an example of the possible trajectories of cities on the clock and we will stick with our US cities example for a bit longer, showing various possible plots and animations.

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From George Kingsley Zipf (1949) *Human Behavior and the Principle of Least Effort* (Addison-Wesley, Cambridge, MA)
The Idea of a Rank Clock—rank is from number 1 at centre to 100 at edge and time goes in years in the usual clockwise direction.

Classic Exemplars: US City Populations 1790-2000

The ‘morphology’ of the clock should tell us something—i.e. the increase in cities, the volatility of ranks and so on.
The rudimentary software for this is on our web site at
http://www.casa.ucl.ac.uk/software/rank.asp
I will show you an animation of the clock but for a long while I simply plotted the clock and left it at that and measured various properties of the dynamics – but this time last year I decided to animate it and of course the work that Ollie and Martin are doing comes from the notion that the graphics need to be livened up.

Here is the US city ranks from my desktop software.

Rank Size Relations for the Top 100 High Buildings in the New York City from 1909 until 2010

*power form (left), log form (right)*

& here is the clock
Rank Clocks of the Top 100 High Buildings in the New York City (a) and the World (b) from 1909 until 2010

There is much more work to do on all this and I am only giving you a taste of this, now back to shape and size.

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**US Metro Area Populations**

My last example – and we have many now – is from the US Bureau of Economic Analysis on metro areas – SMSAs from which I simply took their 366 regions for which population and income data are available for 37 years from 1969 to 2005.

One of the nice things here is that we can plot population ranks etc and income ranks – but we can also look at the ratio – population per capita which is much more volatile than each of the prime variables – let us see.
We can easily plot the shifts, spaces, and clocks for these population and income data. These follow very regular scaling laws, at least in their fat tails. Here is a potpourri

But the real interest is in per capita income/wages – i.e. *Incomes / Population*. How does this rank? And if there are big shifts in rank, this shows divergence of these two variables

As you might expect the rank clock provides a graphic animation of this relative disorder at the micro level
Japanese Populations: Cities at Different Scales

We have a large population data set of 2137 ‘cities’ in Japan that are mutually exclusive subdivisions of the Japanese space and these are not cities in the sense of the US cities we have used.

We also have an aggregation into 269 cities at a higher level.

We also have other attributes of these cities – with data – such as area so we can compute densities.

And we have these from 1920 to 2000 in five year periods.

We will look first at the complete set of counts and then densities for 2137 and 269 and then look at Tokyo: first 2137 for counts and densities.
then 269 for counts and densities

And let us look at the animation of the counts

then Tokyo, counts and densities,
Adding Place to Rank Clocks

Use OpenStreetMap as a base map
Use Google Earth browser plugin for 3D mapping
- Represent each point as an extruded pillar
- Create the pillar size and height based on the extent and average density of data
For simplicity, a uniform distribution of points across a square extent is assumed
Communicate with Google Earth by passing it KML
OpenLayers will readily convert any geometry to KML

Additional Options to Aid Visualisation

Inverting the rank clock
Limiting the size
Colour by:
- Initial interval
- Initial rank
- Final rank
Layering the lines (painter’s algorithm) by:
  - Initial or final highest or lowest ranked

http://splintmap.geog.ucl.ac.uk/~ollie/rankclocks/
Demo – populations in 33 Greater London Boroughs from 1801

Demo – High buildings in London from 1950 to 2015
Demo – Tube Exit Volumes from 2003 to 2009

Demo – Fortune 500 from 1955 to 2010 for a sample of fast rising companies
For United Nations World Cities Population (595 cities) from 1950 to 2025
Animations: Rank Clocks and Rank Space

- Martin Austwick’s has developed another set of programs which are superb at animation using Processing
- The essence of this is to continuously animate the clock and then we can see the deviations
- We can also plot individual trajectories and we can shorten the trajectories to show how each moves relative to one another
- We can do all of this also in rank space but I can’t show you this

Here is the typical user interface and let me show it to you running next page
Next Steps

- **Good measures of the volatility**: some of these I have explored but as yet we haven’t done anything comprehensive – such as distances measures on the clock – measures of spread such as entropy and so on.
- **Extensions to network systems** – trade and migration and traffic flows and their changes over time – in terms of nodal volumes and flows on links.
- **Examining different definitions of cities** – and related systems: cities as exhaustive partitions of space versus cities as nodes or points.  
  *Some papers ..........*
Questions?

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