

*Lecture 6*

# **Underpinning Cellular Automata: Fractal Cities**

<http://www.casa.ucl.ac.uk/ASU/>

# Outline

- What are Fractals? Definitions and Properties
- Scaling and Links to Fractal Patterns
- Fractal Geometries: Patterns and Processes
- City Shapes at Different Scales: Modular Growth
- Fractal Growth Models: DLA
- Applications through Cellular Automata
- Moving to Agents in the Cellular Landscape
- Basic Reading

# What are Fractals? Definitions and Properties

Fractals are objects that scale – they show the same shape at different scales in space and/or time

This property of scaling is sometimes called self-similarity or self-affinity

In our world of cities, we think of this scaling as being a replication of the same shapes in 2 or 3 D

Euclidean space

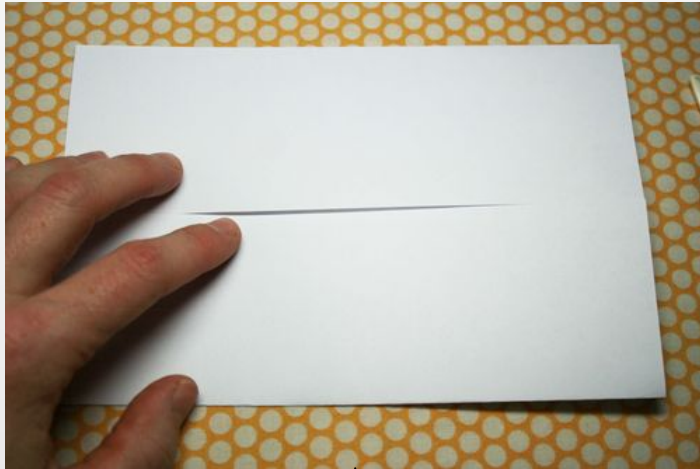
This suggests modularity in growth and evolution and processes that are uniform over many scales

The signature of a fractal is called its dimension and usually this suggests how the fractal fills space. If we think of 0-d as a point, 1-d as a line, 2-d as a plane and 3-d as volume, then a fractal also has fractional dimension.

This means that the Euclidean world is the exception not the rule as the integral dimensions are simplifications.

The best example of a fractal is a crumpled piece of paper

It is 2-d but when we crumple it we make it more than 2-d



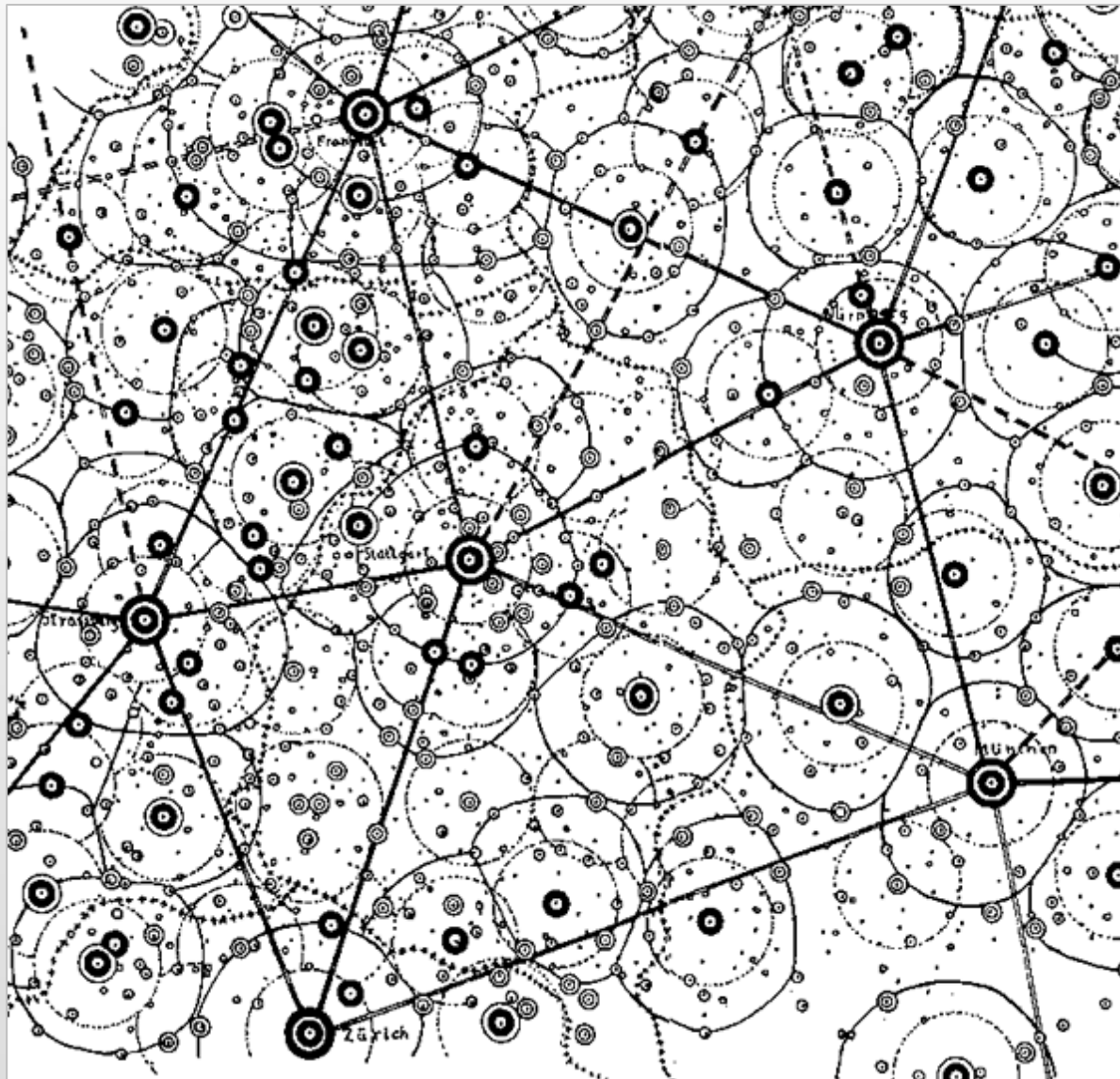
Other great examples are tree structures ....

## Scaling and Links to Fractal Patterns

In fact in mathematics a function is scaling if it can be shown to be scalable under a simple transformation – i.e. if we can scale a distance by multiplying it by 2 say and the function does not change qualitatively, then it is scaling – so power laws – functions like  $f(y)=x^{-1}$  scale because if we multiply  $x$  by 2, say, we get  $f(2y)= (2x)^{-1}=2^{-1}x^{-1}\sim f(y)$

We will not take this further but just point out that rank-size, even exponential functions imply

fractality – see the web site and the pdfs on scaling and entropy and fractals. In other words, if we take away space from our models, then what is often left in fractal phenomena is the idea that the aggregate scales in fractal terms. Good examples of this are in terms of central place theory – in the order between big centres and small centres e.g.

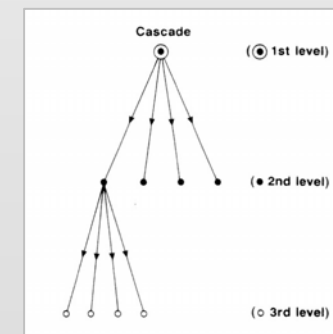
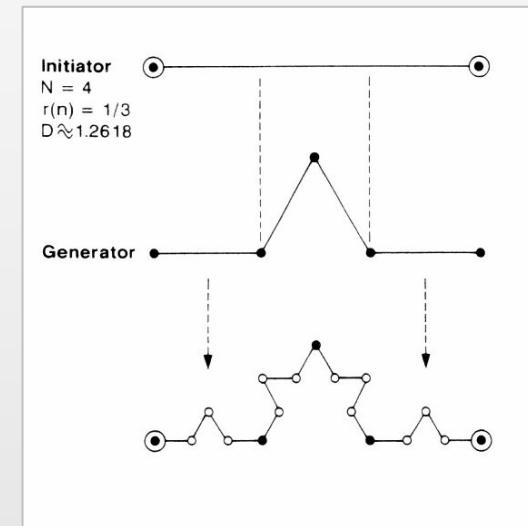
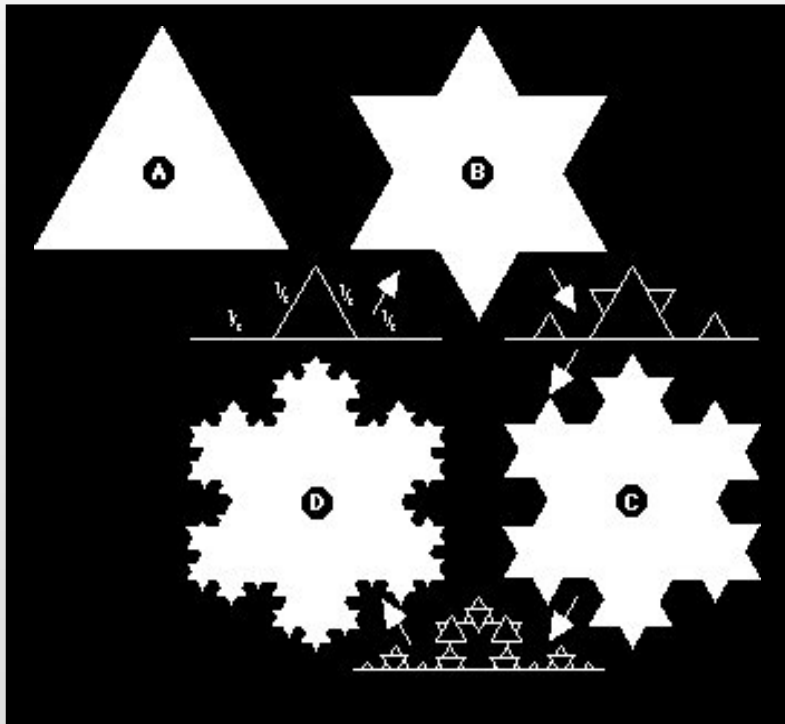




# Fractal Geometries: Patterns and Processes

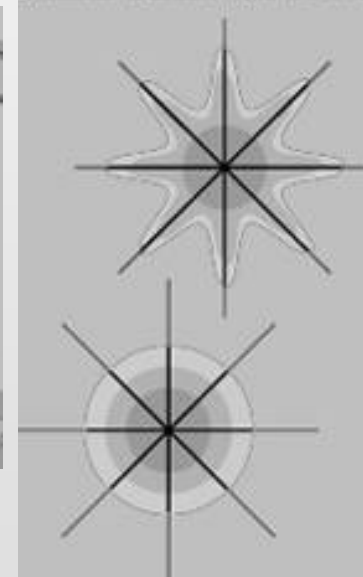
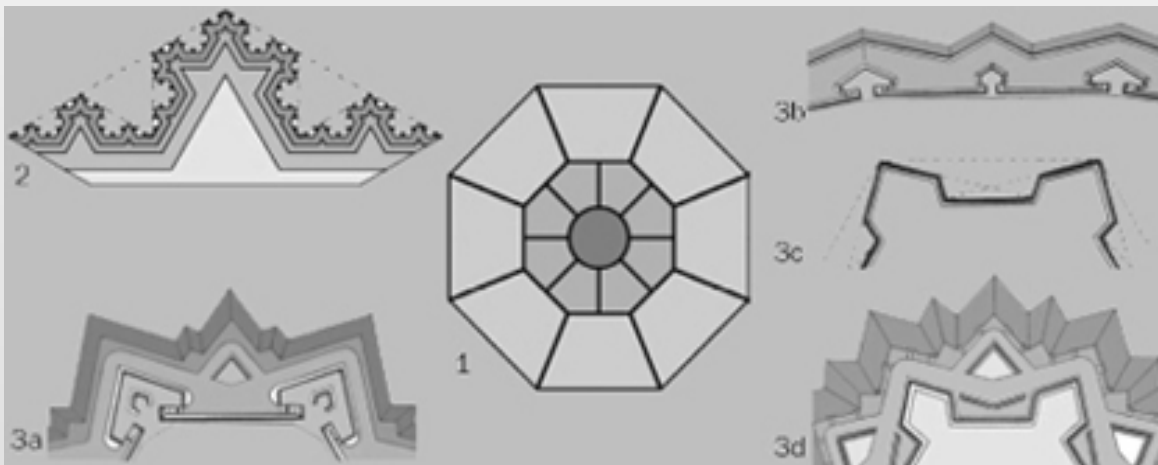
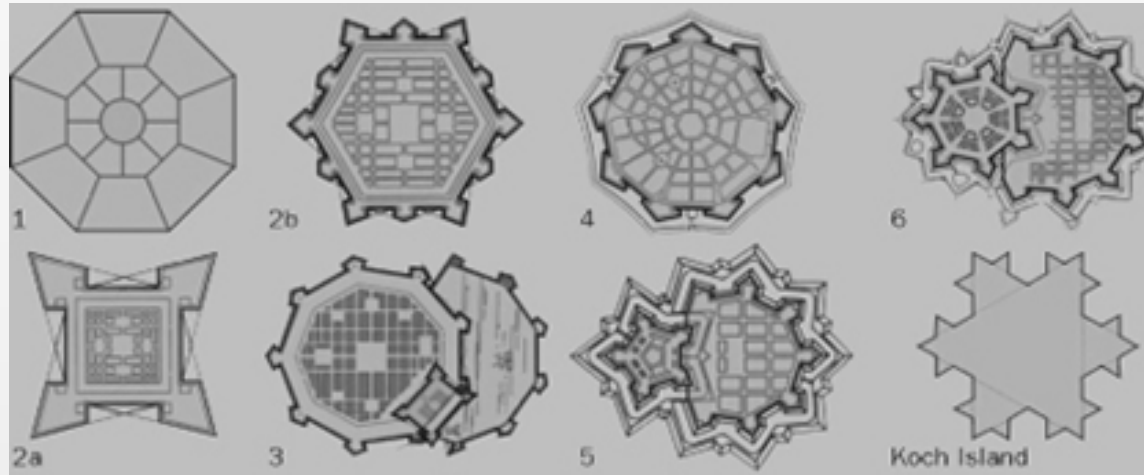
There are some basic conundrums and paradoxes with fractal geometry – the clearest one is the length of a fractal line – if a line is truly fractal, it fills space more than the line and less than the plane with a fractal dimension between 1 and 2. As it also scales – any bit of it has the same shape as an enlarged or reduced bit but the length is infinite. Note the famous paper in Science in 1967 by Mandelbrot – *How long is the coastline of Britain?*

We can show this for the Koch curve. Note how we construct the irregularity by adding a scaled down piece of the curve



Note how hierarchy is a feature of the construction

And note how the line is infinite but the area is finite



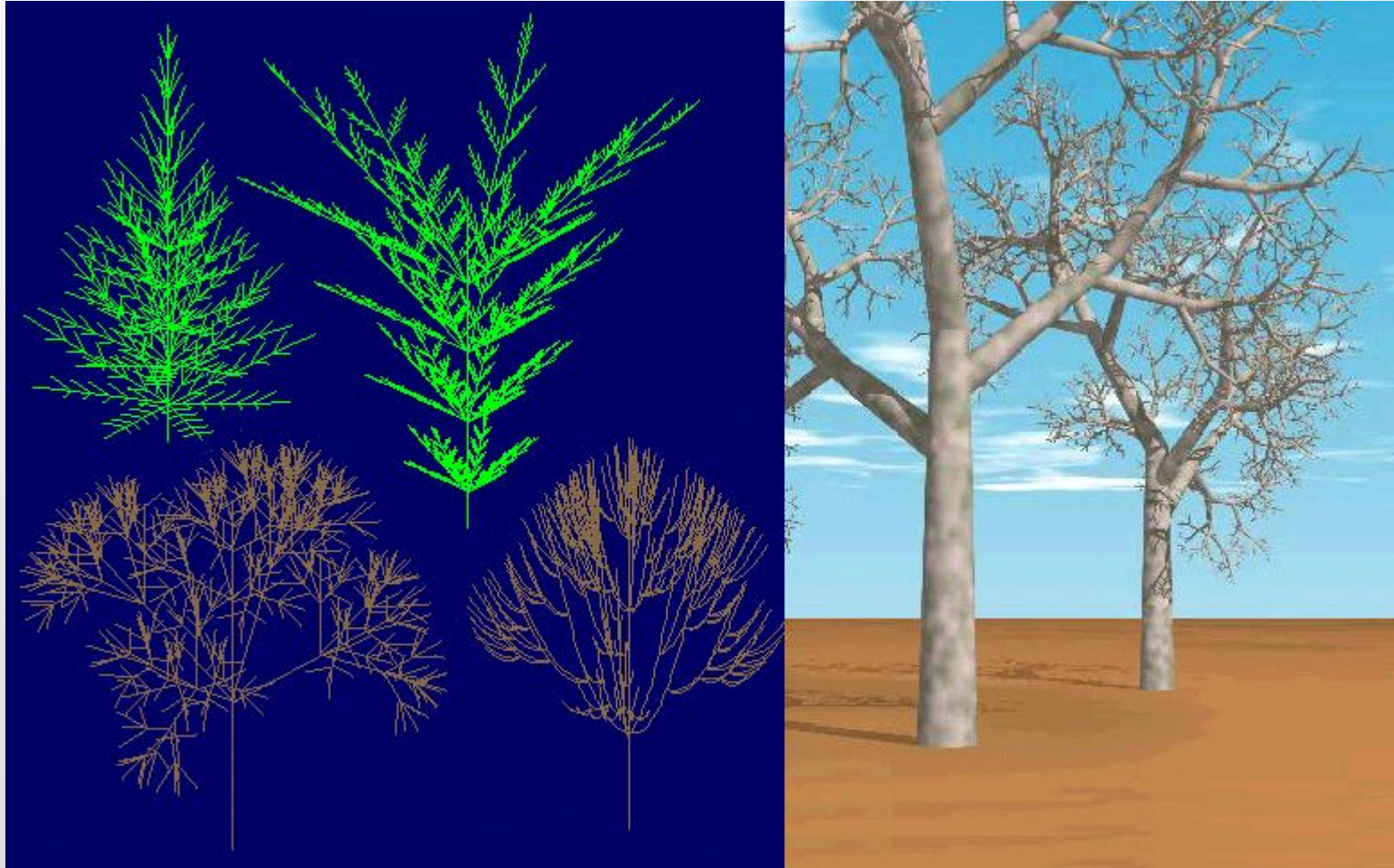
This is resonant in ideal towns and  
In many shapes in nature as we show ...



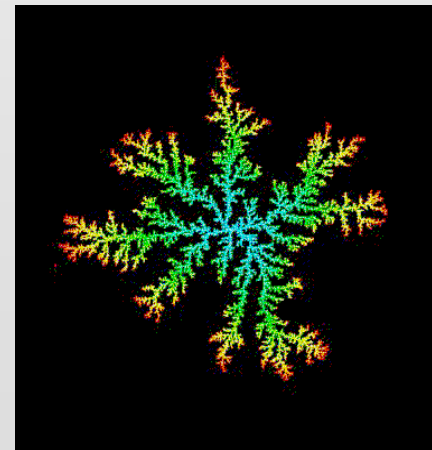
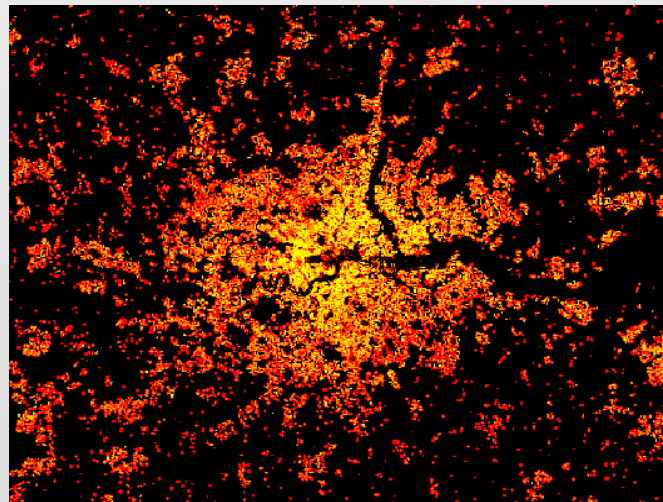
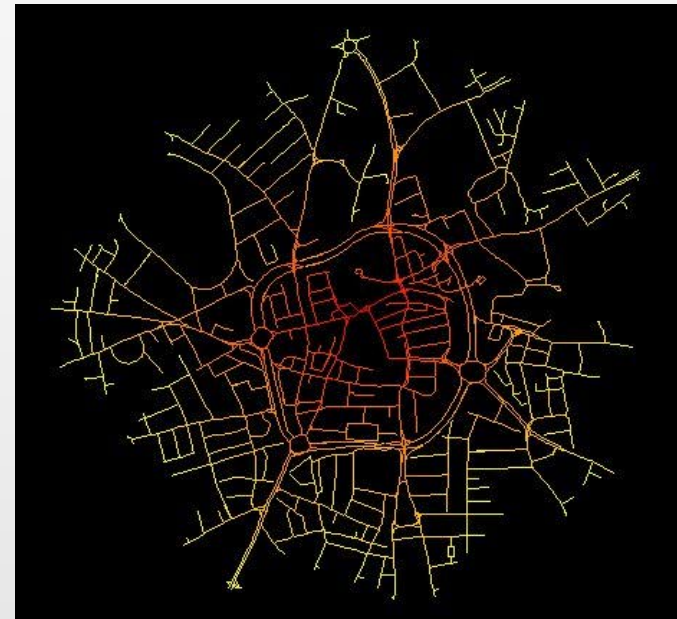
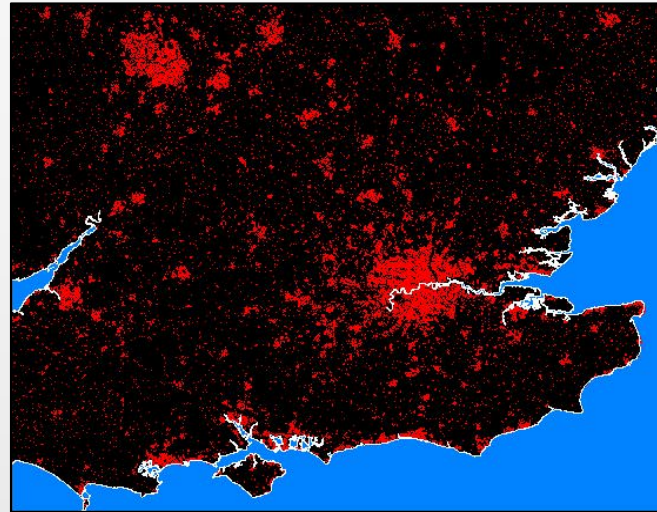
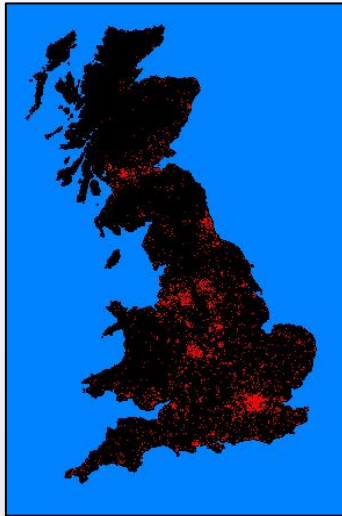
Barnsley's fern, from his book Fractals Everywhere which is generated by a rather sophisticated mathematical systems of routine and repetitive transformations called the Iterated Function System



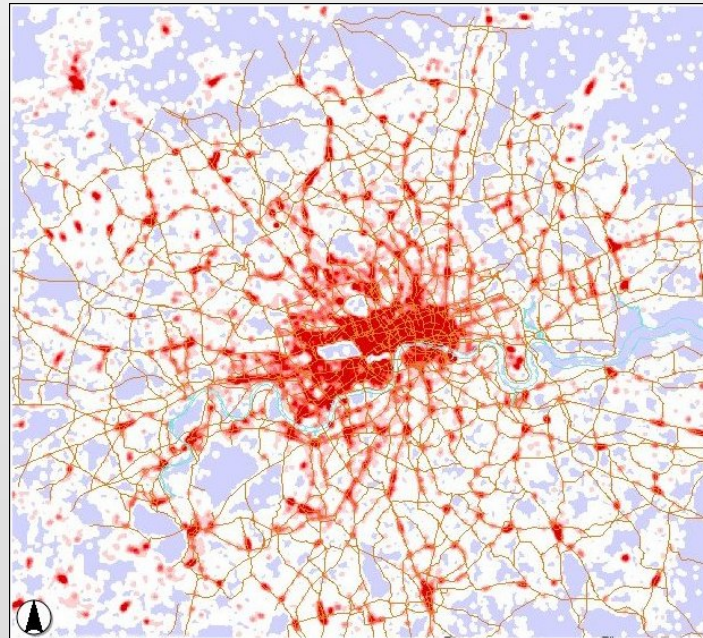
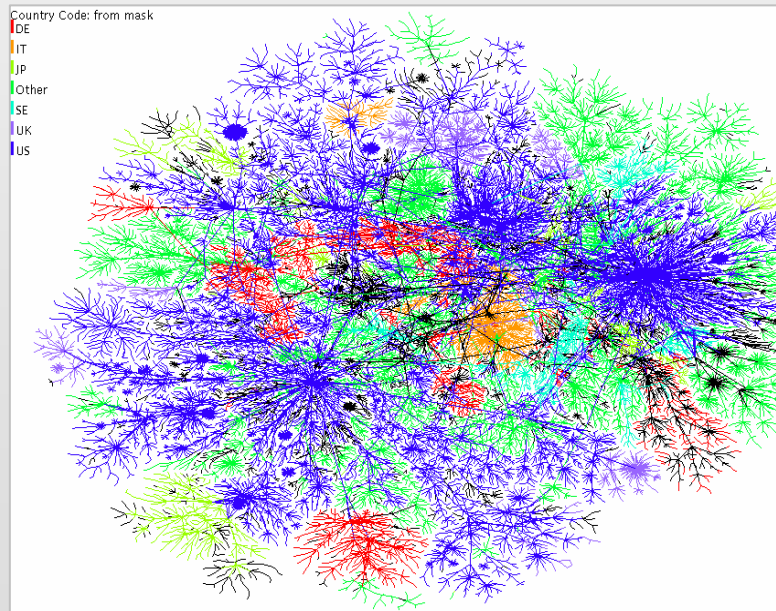
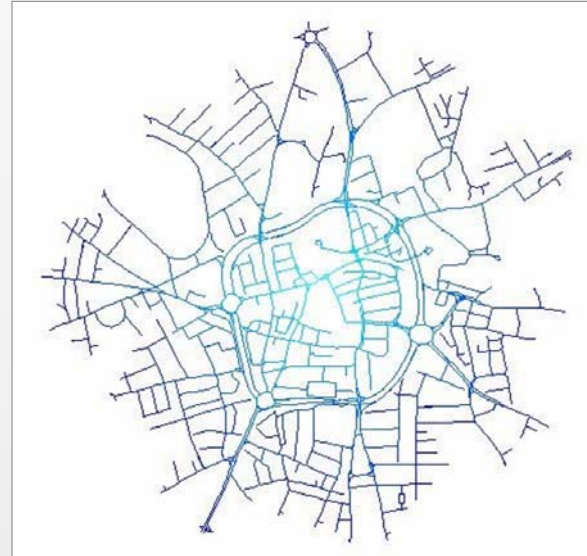
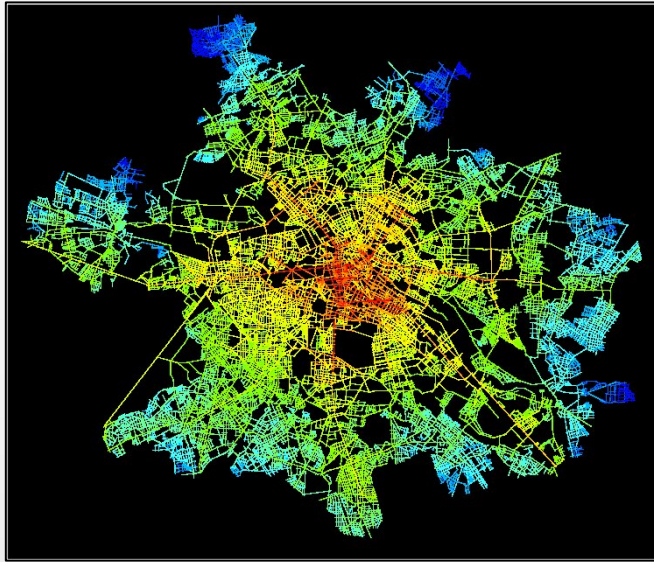
Computer graphics depends upon fractals ! At least for natural forms such as trees



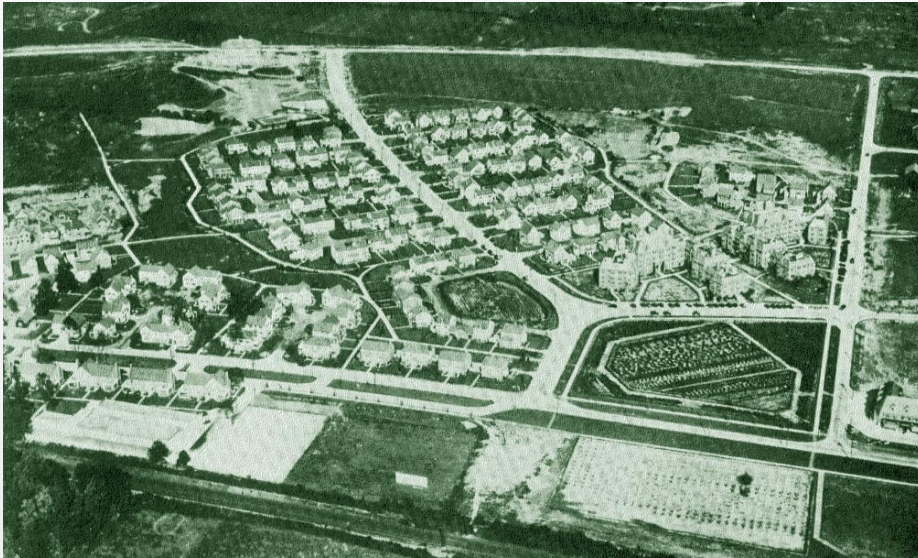
# City Shapes at Different Scales: Modular Growth



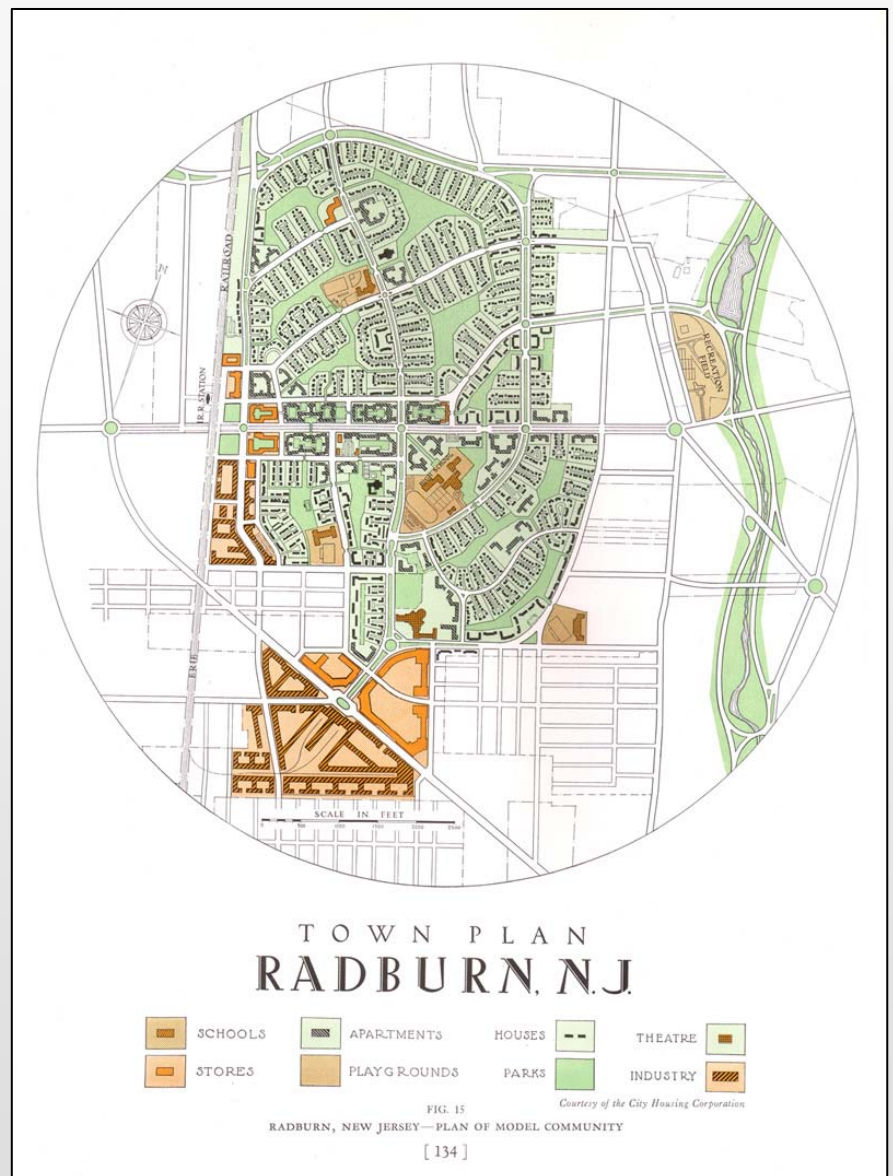
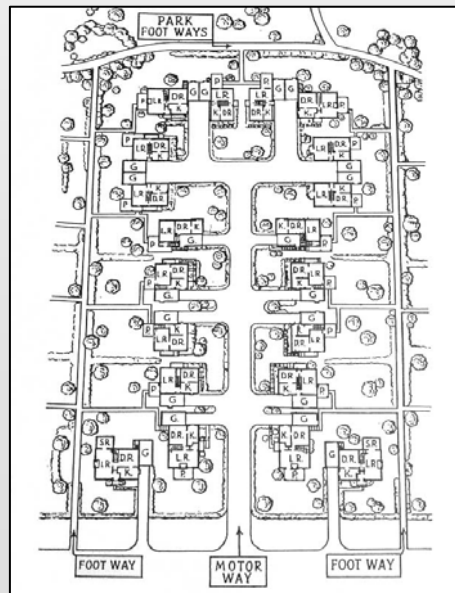




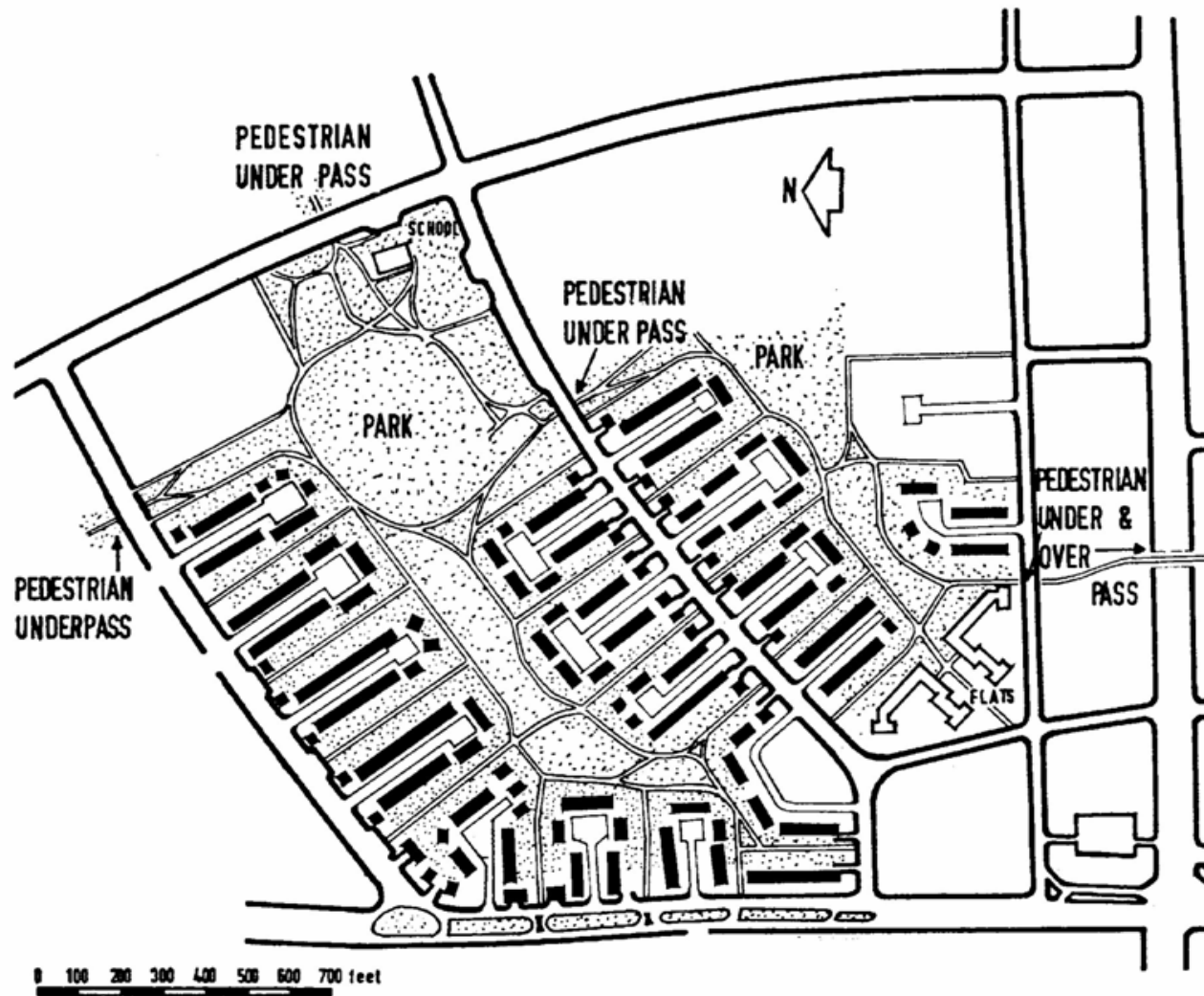




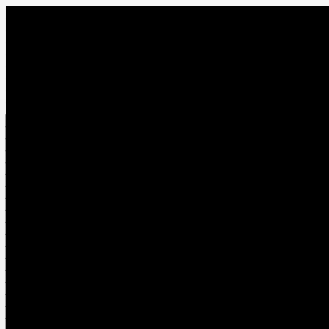
An early 'new town'  
RADBURN, NJ  
1920s



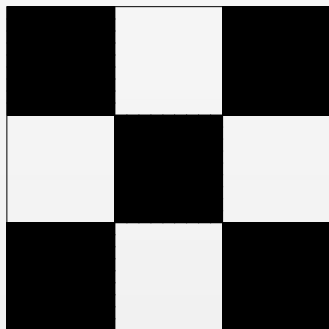




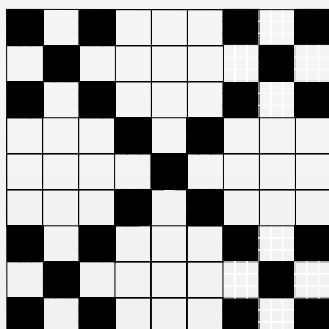
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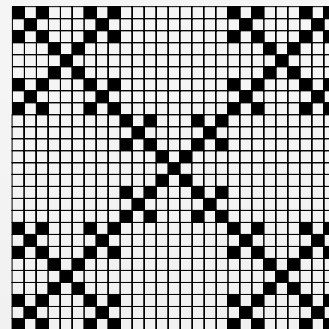
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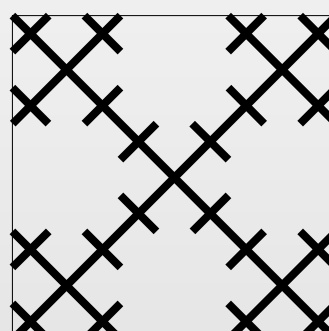
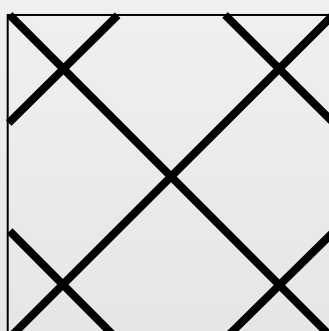
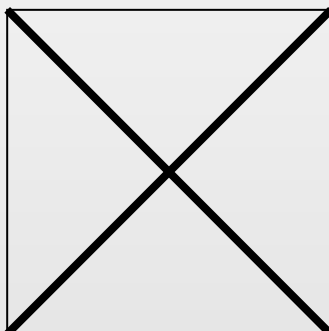
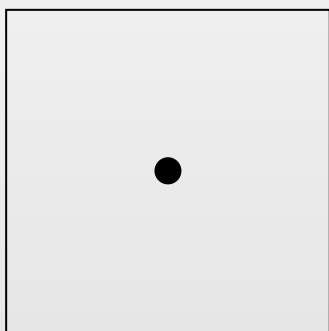
$k=2$



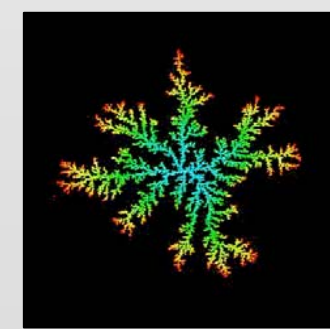
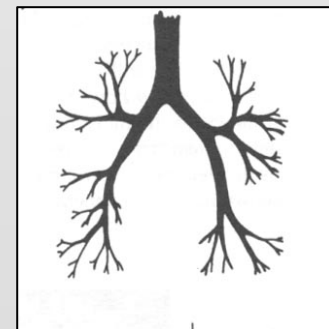
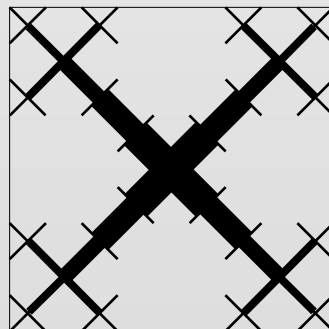
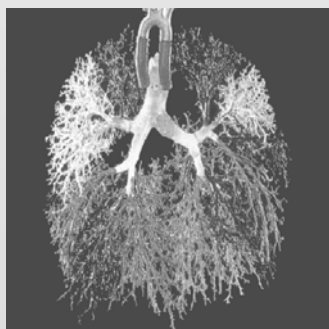
$k=3$



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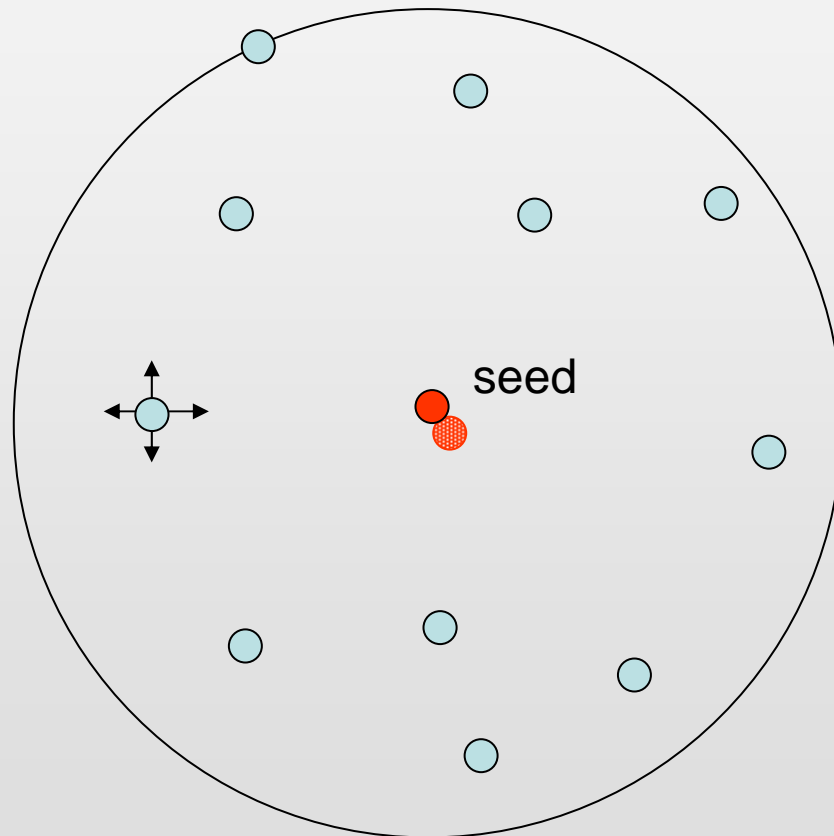
# Fractal Growth Models: DLA

Ok, let me show you the simplest possible model of an organically growing city – based on two simple principles

- *A city is connected in that its units of development are physically adjacent*
- *Each unit of development wants as much space around it as it needs for its function.*

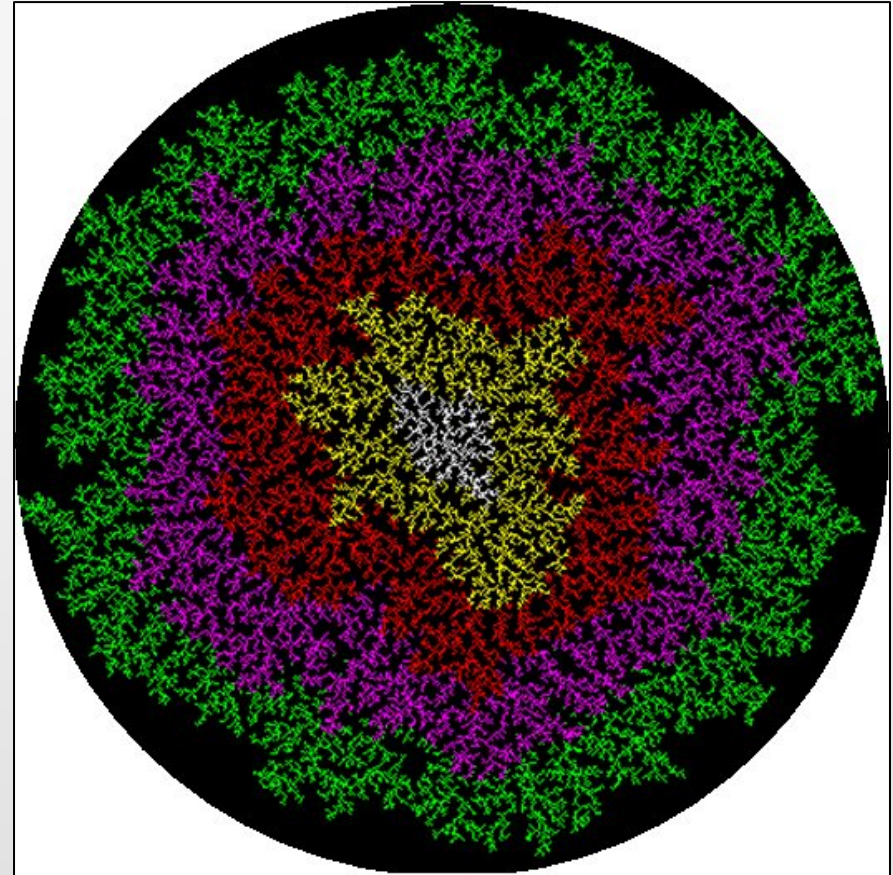
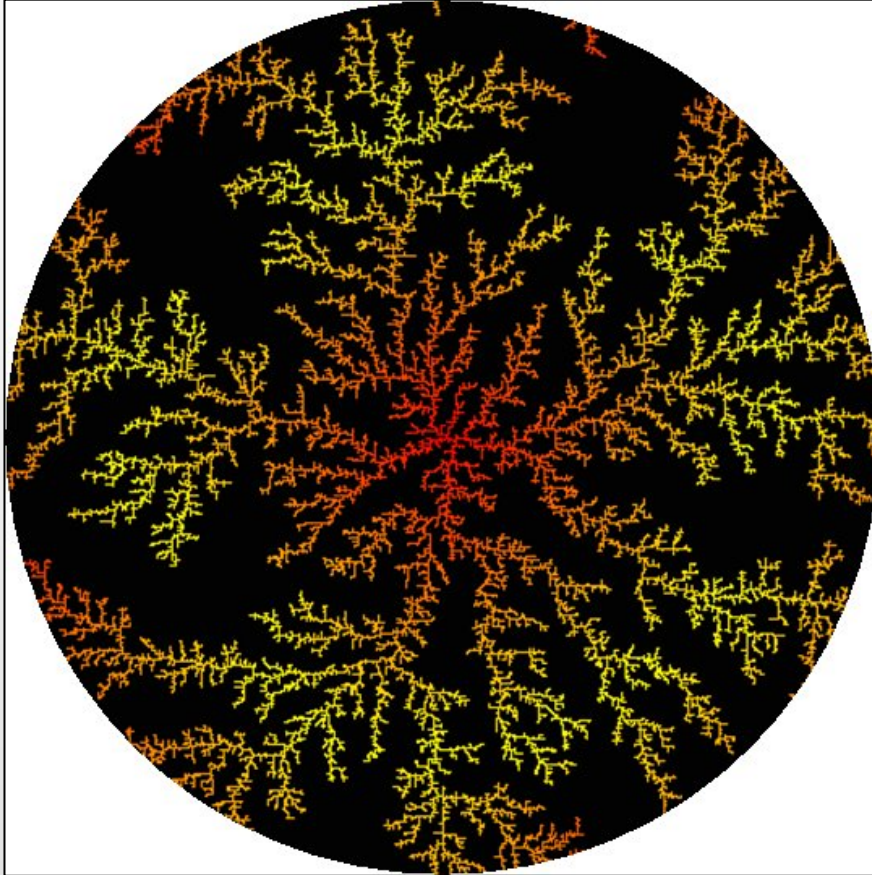
We start with a seed at the centre of a space and simply let actors or agents randomly walk in search of others who have settled. When they find someone, they stick. That is all.

In essence, this is random walk in space which can be likened to the diffusion of particles around a source but limited to remain within the influence of the source – the city



We can run a little program to show this. I did this in the first lecture as an example of toy model





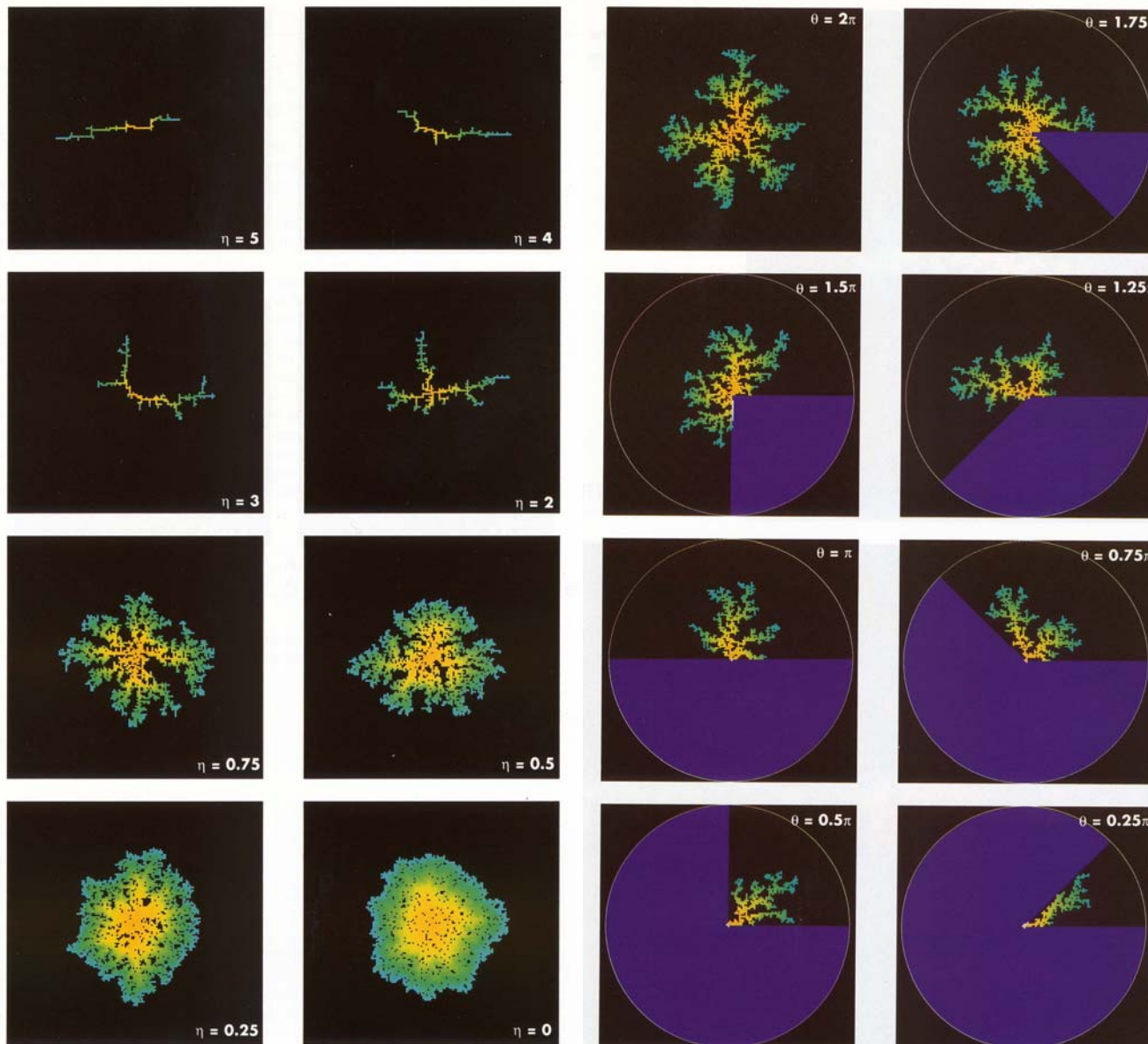


Plate 8.4 Urban Forms Generated by Systematic Distortions to the DBM Field

Plate 8.2 Physically Constrained DBM Simulations



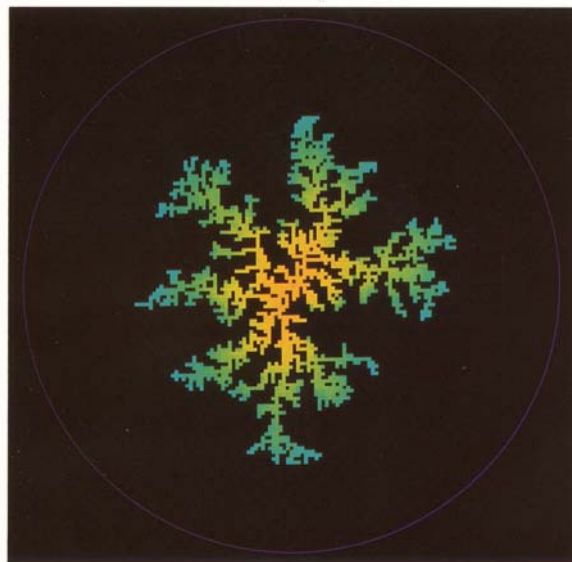


Plate 8.3 (left) The Baseline Simulation  $\eta = 1$

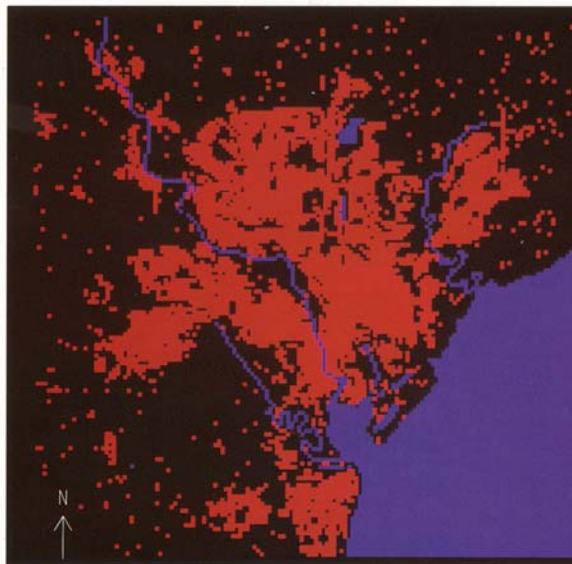


Plate 8.5 (below) The Urban Area of Cardiff.

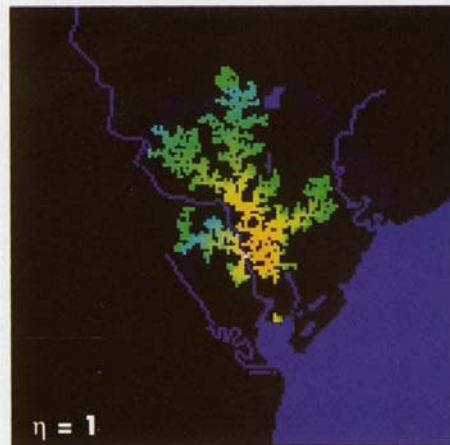
# CARDIFF URBAN GROWTH

Digitised From  
1:50000 OS Map  
Sheet 171

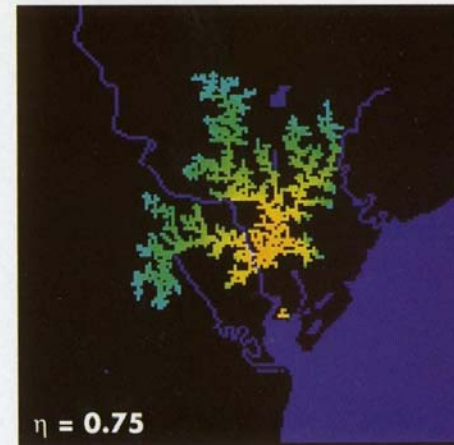
SYSTEM HISTORY  
NOT AVAILABLE

Urban  
Development  
Rivers and  
Estuary

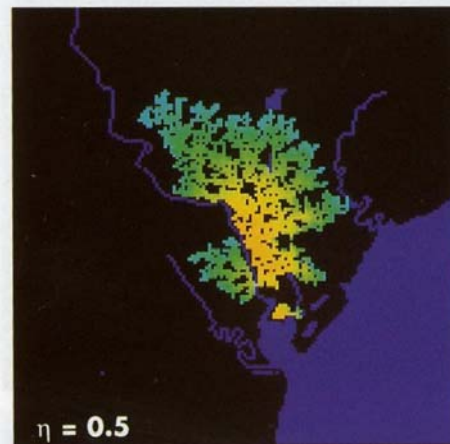
0 Kms 2



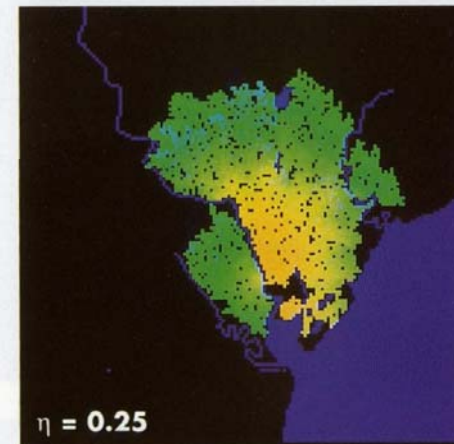
$\eta = 1$



$\eta = 0.75$



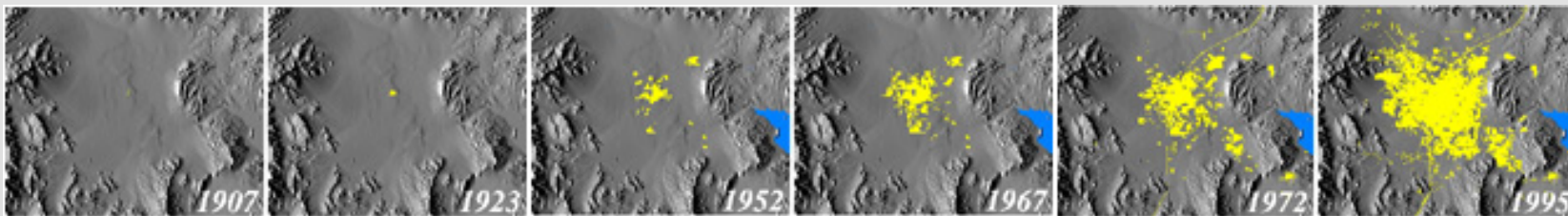
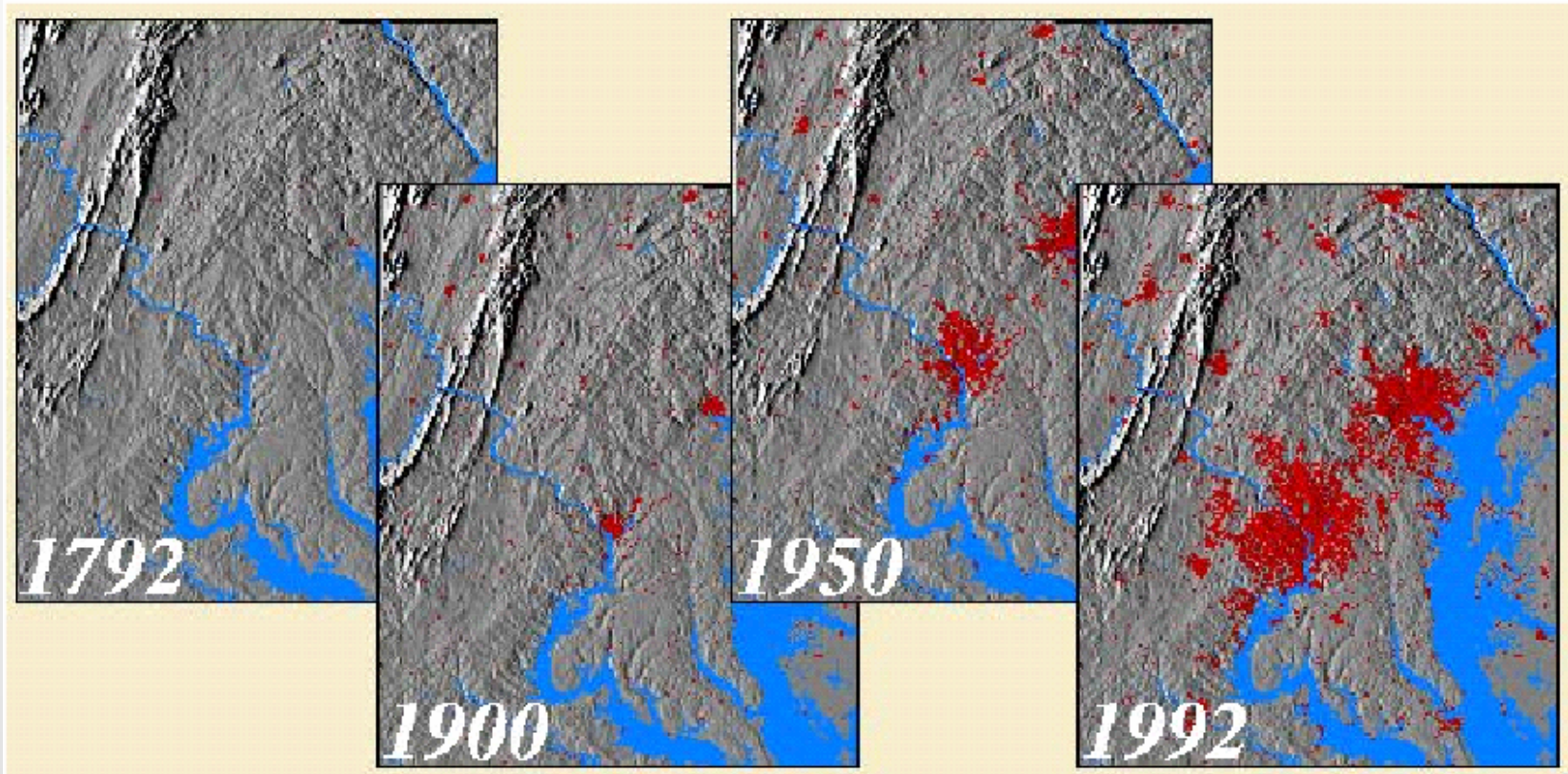
$\eta = 0.5$



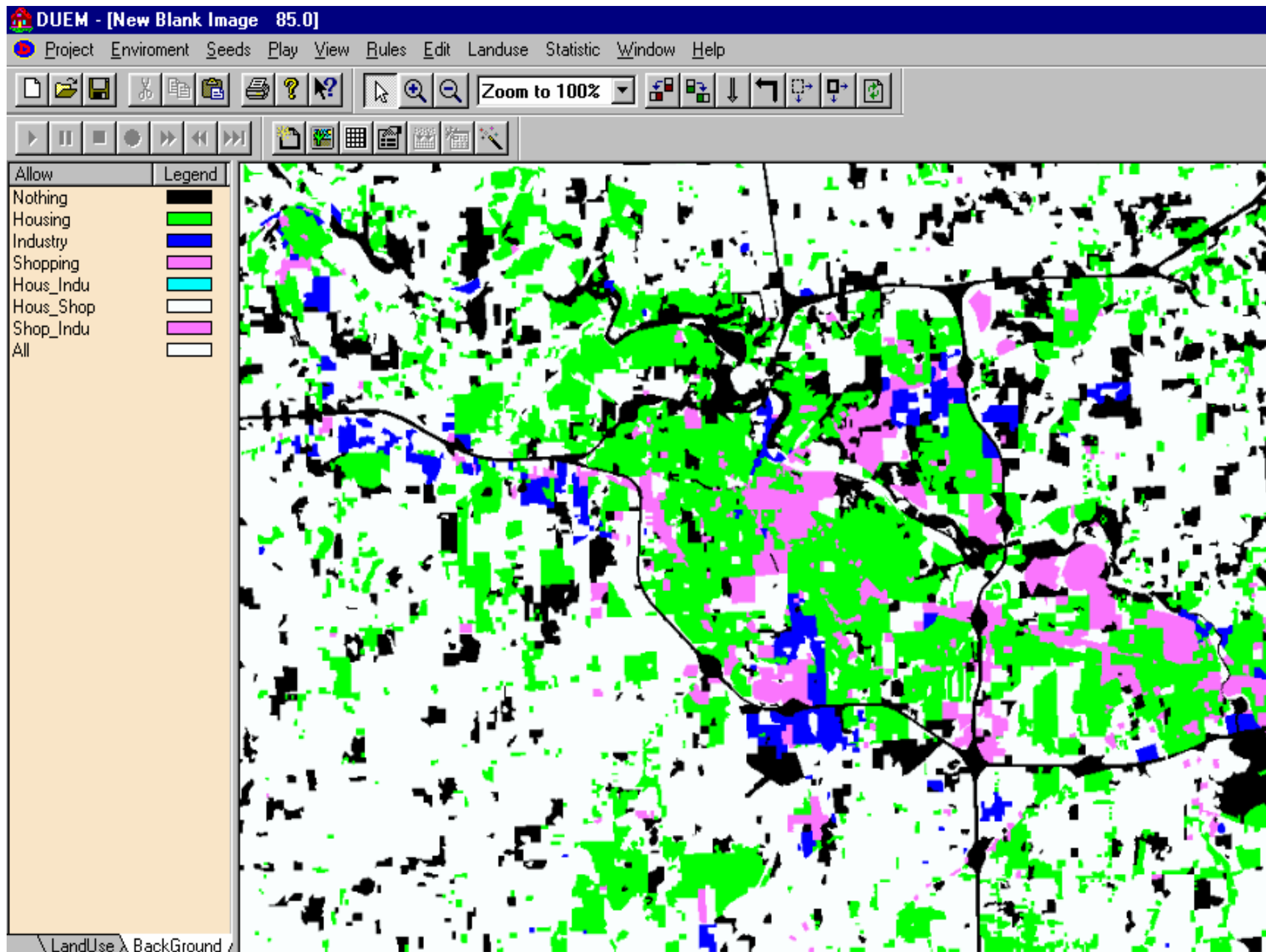
$\eta = 0.25$

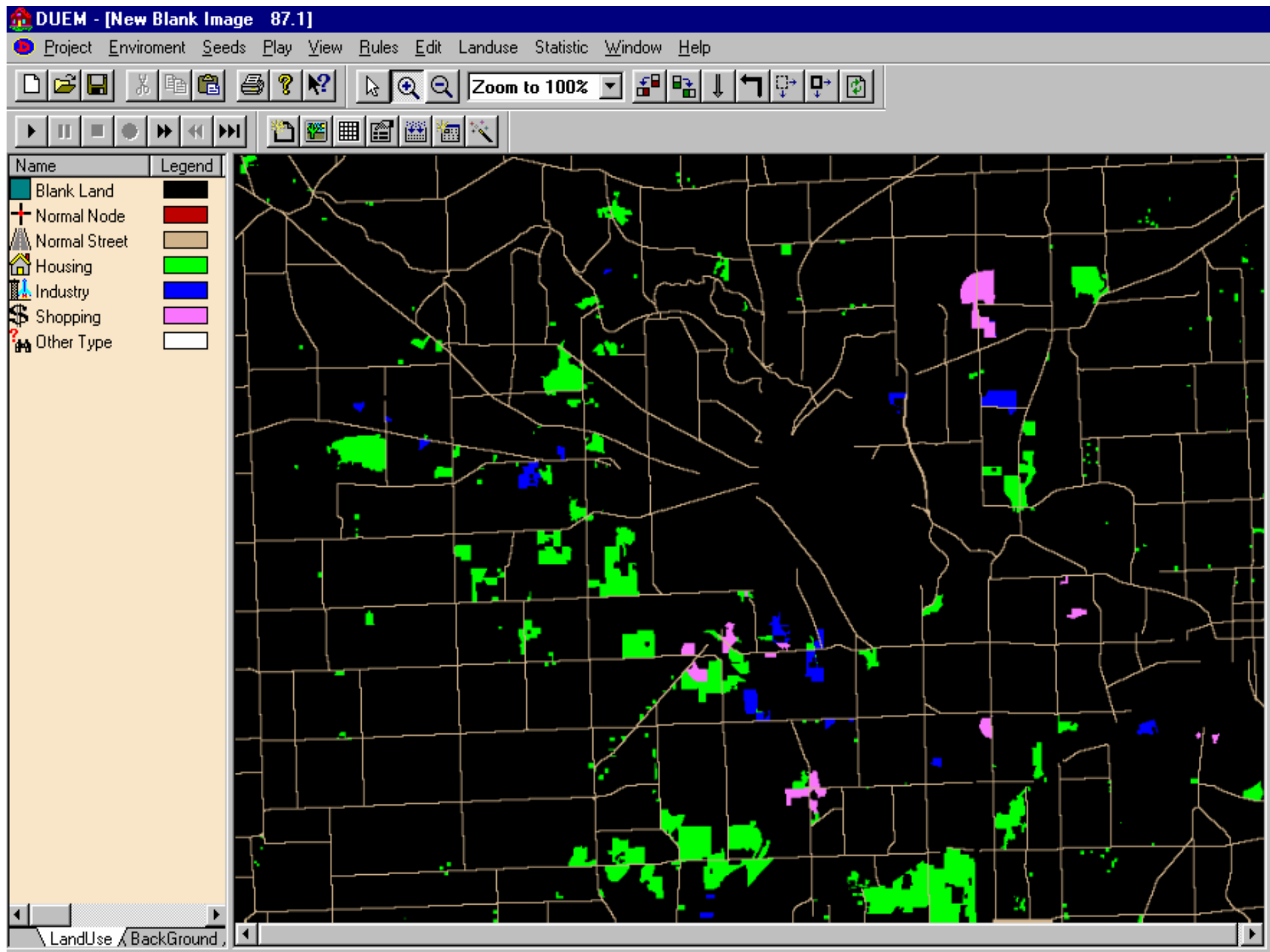
Plate 8.6 Simulating the Urban Growth of Cardiff

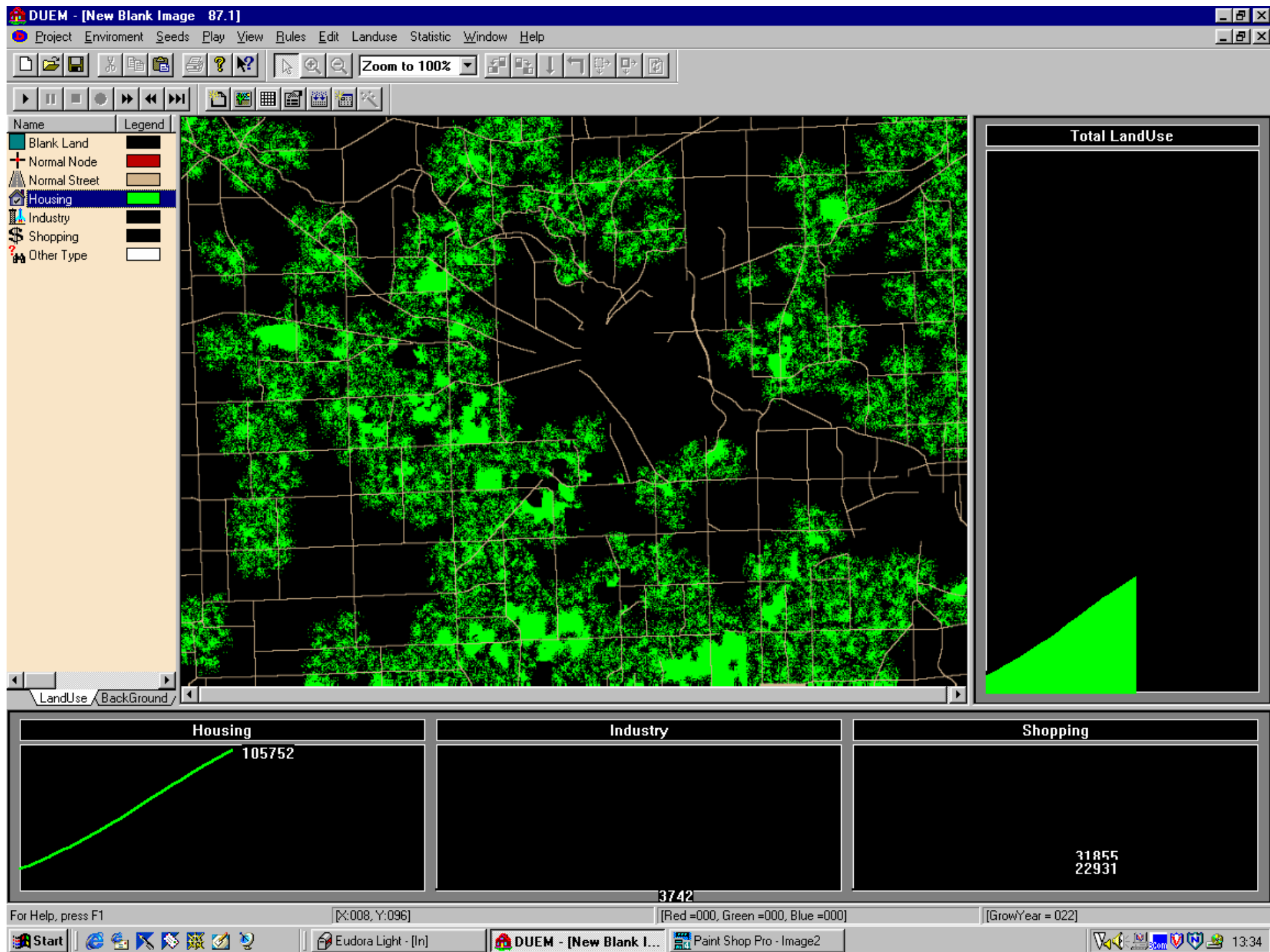
# Applications through Cellular Automata



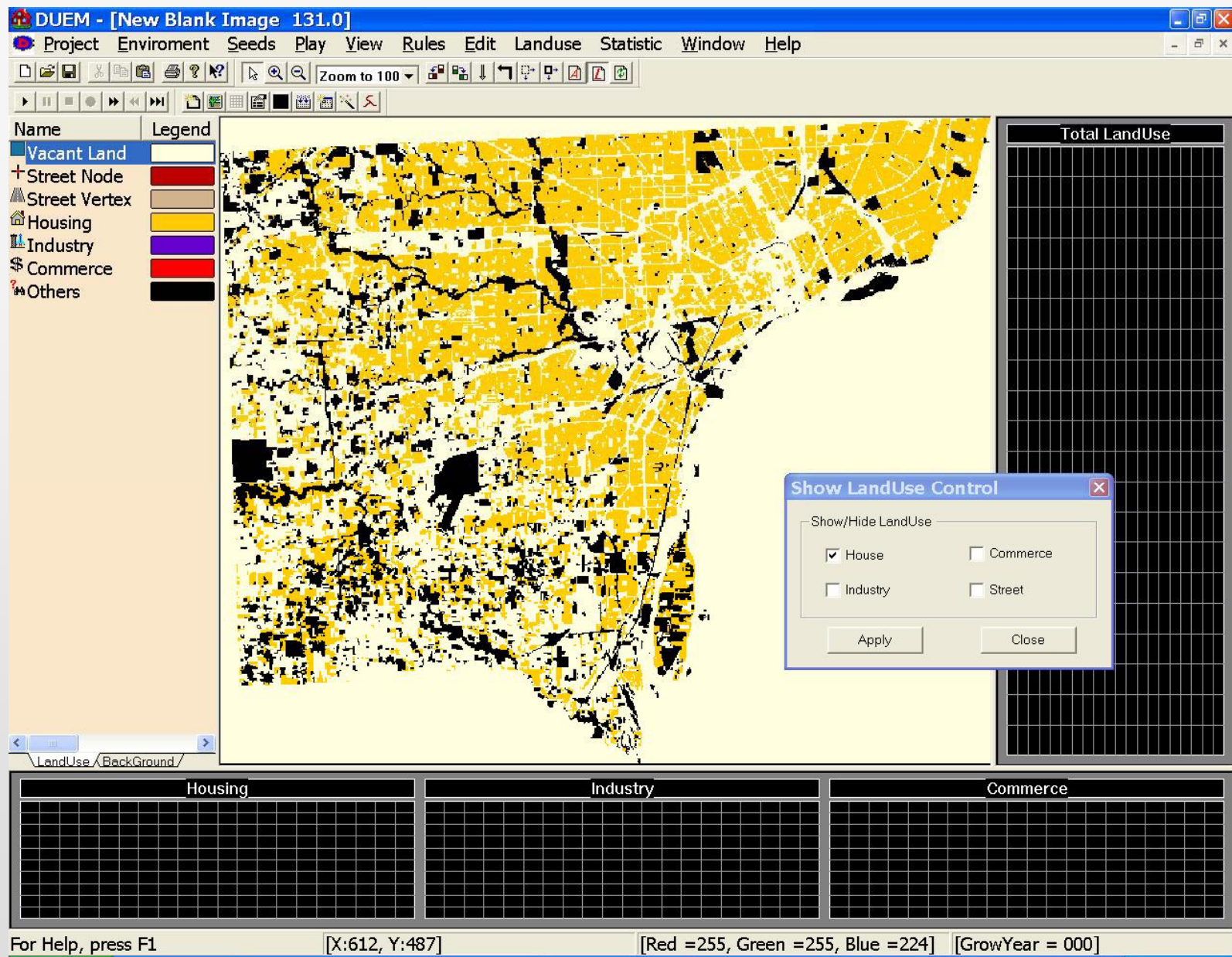




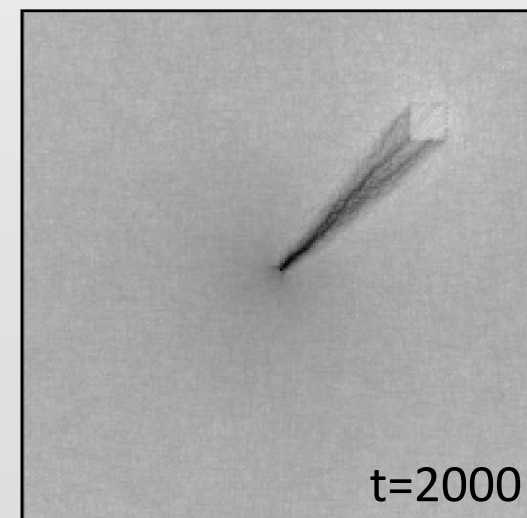
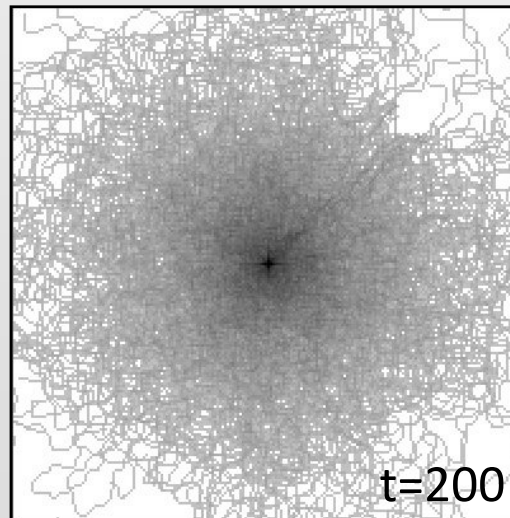
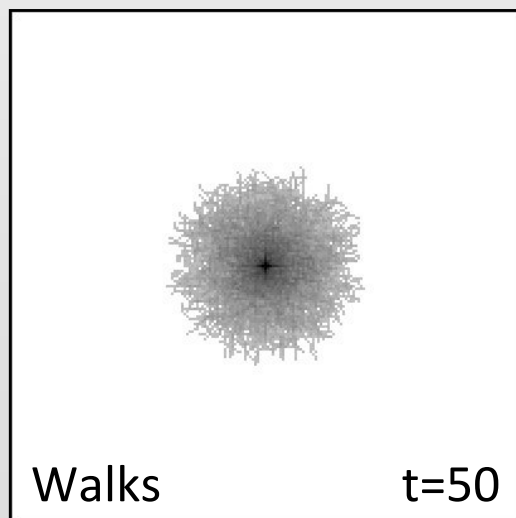
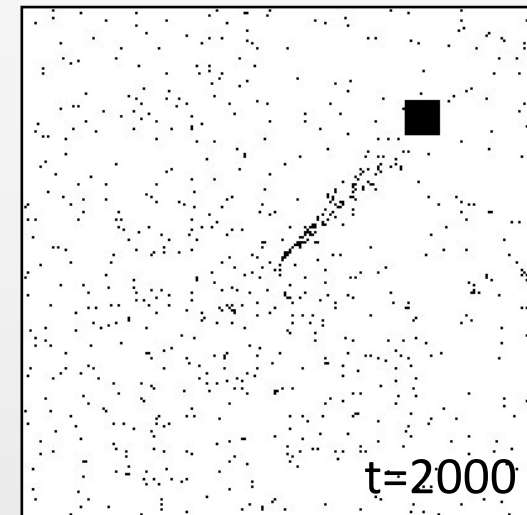
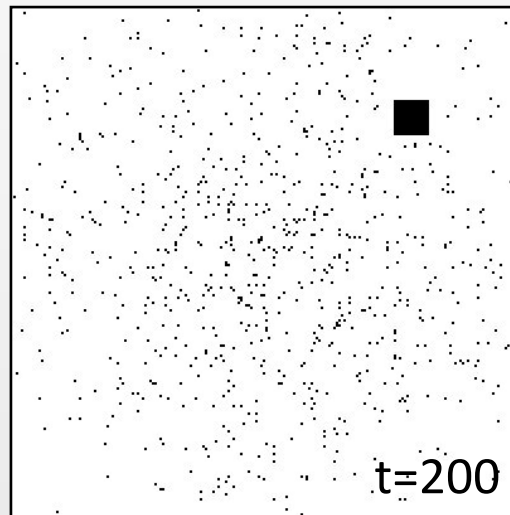
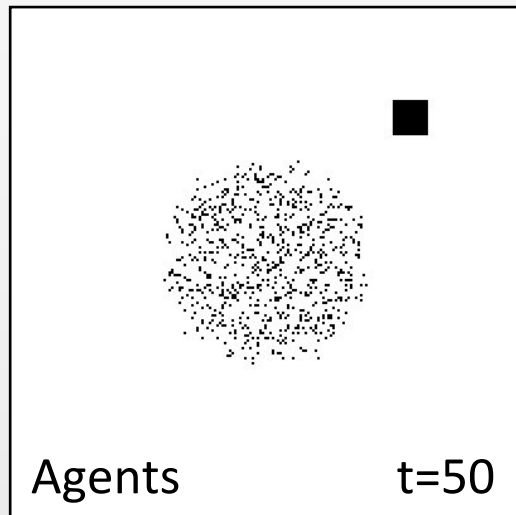




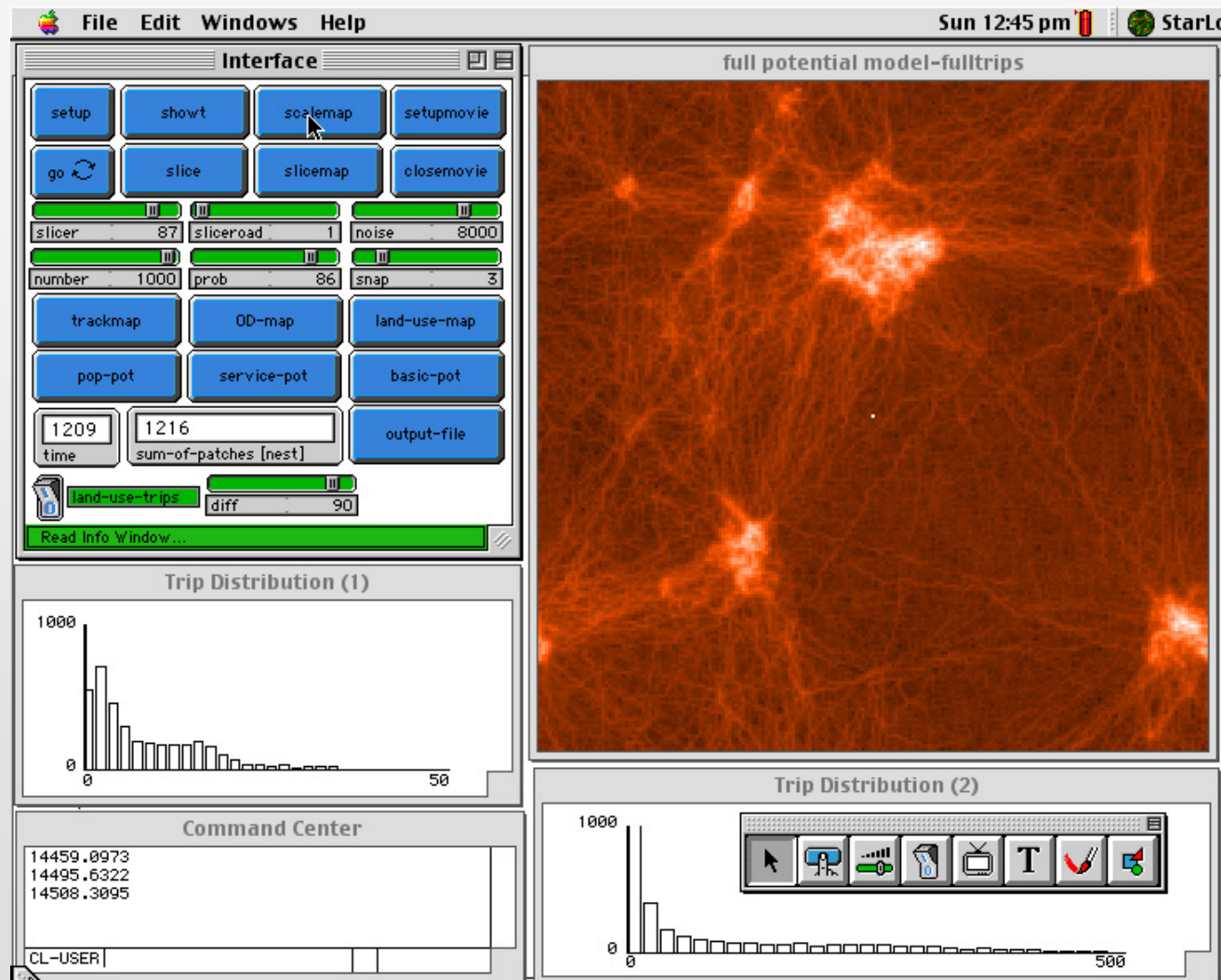




# Moving to Agents in the Cellular Landscape



# A Typical Visual Interface for these Agent-Based Models



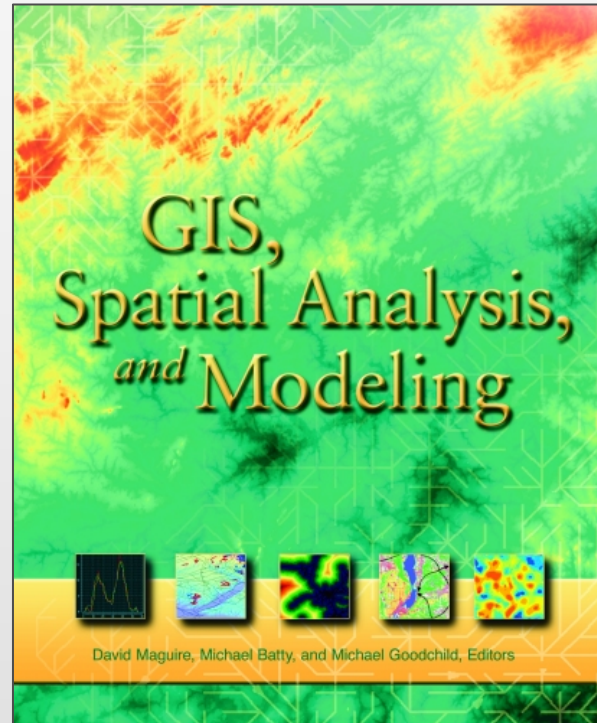
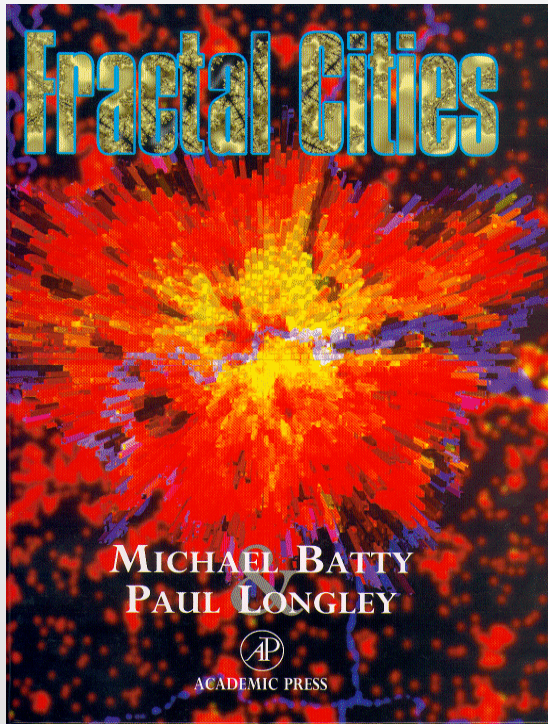
## Basic Reading

I don't have time to get into how we can build fractal models with agents in any detail but I refer you to my book *Cities and Complexity* in one of the middle chapters – 5 or 6 I think for an elaboration of how we can link agents to fractals – link CA landscapes to agents.

At this point, we have run out of time but let me point you in the direction of some reading for this last talk today



My book **Fractal Cities** with Paul Longley is online at [www.fractalcities.org](http://www.fractalcities.org). And you can download it



There are some nice articles in the edited book by Maguire et al. on not only CA and ABM but also LUTI models too



# Reading

I will put material up on the web tomorrow

***Any Questions?***

[www.casa.ucl.ac.uk](http://www.casa.ucl.ac.uk)